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A Study of the Effects of Marketability on the Distribution of Market Generated Returns.

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A STUDY OF THE EFFECTS OF MARKETABILITY ON THE DISTRIBUTION OF MARKET GENERATED RETURNS

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ABSTRACT

Very little theory has been developed on the effect of marketability on the distribution of returns. As a result, this study is an emperical exploration of that relationship, without any strong, preconceived hypotheses.

Since the distributions were likely to be non-normal, but otherwise indeterminate as to form, a test had to be developed to determine the existence of differences. It was decided that the first four moments and the studentized range could effectively capture the characteristics of a distribution and provide a set of measures to serve as variables in discriminant analyses.

Marketability was defined as shares traded divided by shares outstanding. Four samples were selected representing differing degrees of marketability, but homogeneous in all other respects. These samples were then rigorously tested on the basis of both daily and monthly holding periods. The results of these tests indicated that a daily holding period is too short to reveal any reliable results. The analysis of daily returns produced results that conflicted with any logical risk-return relationship and that were inconsistant with the results of the tests conducted on the monthly holding period sample. The tests using the monthly holding period sample did indicate a significant relationship between marketability and the characteristics of ex post market generated return distributions.

Further testing was conducted on random portfolios generated from the samples, verifying the prior results and indicating a strong non-diversifiable component in the relationship. Differences in marketability did not influence the speed of diversification.

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Chapter I

INTRODUCTION

This investigation is to determine whether significant differences exist in the statistical characteristics of the daily and/or monthly return distributions of New York Stock Exchange (NYSE) and American Stock Exchange (AMEX) listed stocks as a result of the marketability of the issue. Very little literature, either theoretical or empirical, has addressed this question. Of the two major empirical studies, one suffered from design problems[1] while the other relied on a sample composed of just four indexes.[2] Thus there is a lack of convincing evidence about the relationship between return distributions and marketability.

As an extension of the existing empirical work, this study (1) develops single and multiple discriminant models for testing differences in group performance for four levels of marketability, (2) classifies samples of daily and monthly returns using the models that were developed, (3) creates a series of portfolios for both samples to reduce the

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[1]Kalman J. Cohan, Walter L. Ness, Jr., Hitoshi Okuda, Robert A. Schwartz, and David K. Whitcomb, "The Determinants of Common Stock Returns Volatility: An International Comparison," Journal of Finance, 31:733-740, May 1976.

[2]Andrew J. Senchack, Jr., and William S. Barnett, "Price Behavior in a Regional Over-the-Counter Securities Market," Unpublished Working Paper, March 1977.

impact of unsystematic factors, (4) builds a new set of discriminant models based on the portfolios, (5) classifies the portfolios using the models, and (6) analyzes graphically the mean levels of each of the characteristics for the four marketability levels.

For purposes of this study marketability is defined as the percent of outstanding shares traded over a period of time. An annual value computed for the year 1978 is used.

The study combines data from the NYSE and AMEX. These two markets have nearly identical mechanisms and draw their participants from essentially the same investor population. Combining data from the two exchanges provides a larger population from which to draw the sample. The over-the-counter (OTC) market is excluded because of the lack of available data and the differences in its trading mechanism when compared to the organized exchanges.[3]

Methodology

The purpose of this study is to test the hypothesis that marketability affects the observed distribution of security returns and, by inference, the return generation process of stocks listed on the NYSE and AMEX. This testing takes the form of several univariate and multivariate analyses conducted on samples of daily and monthly holding period return. The daily sample covers 1978 while the monthly sample covers the same securities over the period of July 1974 thru June 1979.

[3] Several other elements related to the selection of the data samples are examined in Chapter III.

Four groups of one hundred stocks each are analyzed initially on the basis of two four-factor multiple discriminant models. One model is constructed from the first four moments of the return distributions of each stock. The second substitutes the studentized range for the fourth moment. These two models are constructed for daily and again for monthly holding period returns. Although these models will not be able to isolate the specific causes of any differences that may be found, they will determine if any differences exist. Assuming the groups can be discriminated to an extent that would indicate the presence of a marketability factor in the return distributions, a more detailed analysis of univariate discriminant models constructed for each of the five distributional characteristics is pursued.

The second phase of the analysis removes unsystematic influences from the distributions by constructing naively diversified portfolios from each marketability group. The portfolios range in content from one to thirty securities. Equal dollar weighting is used in all cases. To provide a sample size sufficient to produce reliable results, twenty non-identical portfolios are constructed for each marketability group and portfolio size. The portfolios are then used to examine each combination of parameters for both the daily and monthly samples.

Two types of analysis are applied to the portfolio samples. Discriminant analysis is used to determine the significance of any differences that are found. Graphical analysis is also used to illustrate the ordering and determine the consistancy of ordering of any differences that are found.

These two techniques provide insight into the effect of marketability on diversification and the return generation process. By studying the progression from the one-security portfolios one may evaluate the effect of marketability on the diversification process. Fisher and Lorie have suggested that for randomly selected NYSE stocks, portfolios of sixteen to twenty stocks eliminate ninety percent of the diversified variance.[4] If diversification is not seriously impeded by low marketability, the systematic portion of any differences that exist can be examined by studying the portfolios composed of twenty or more stocks.

Limitations on Research Effort

This study, as with any research effort, must operate within a set of boundaries and must recognize that it is not possible to deal with every issue that surrounds the central topic. From the outset a number of limitations should be recognized.

1. Only common stock returns are used. The same questions could be raised relative to preferred stocks, bonds, or any other publicly traded security. Each of the other securities could form the basis for a complete study.
2. Only one measure of marketability is used even though several others have been suggested in the literature. It is very possible that other measures would produce different results and different interpretations. Because of the diversity in the questions that are being addressed, the examination of alternative measures of marketability would have generated confusion and distracted from the central focus of the study.

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[4] Lawrence Fisher and James Lorie, "Some Studies of Variability of Returns on Investments in Common Stocks," Journal of Business, 43:99-134, April 1970.

3. The only stocks considered for inclusion are those for which price and dividend data for the entire period of study could be obtained in machine readable form. The massive amounts of data necessary to analyze a specific stock precluded the use of any stocks that would have required manual data collection.
4. Stocks traded over-the-counter or on regional exchanges are excluded. The market mechanism for trading these stocks is considerably different than what is used by the New York or American Exchanges. Mixing the market mechanisms would have lead to potential biases in the results. The limitations imposed in item 3 above made it impossible to carry out the entire study using only over-the-counter stocks.
5. All available stocks are not used, but rather a stratified random sample is utilized. Marketability is a reasonably continuous measure when viewed over the population of available stocks. The only way differences can be detected in such a situation is to stratify the population and select non-adjacent strata to form the sample. In this study, each strata contains 100 stocks. The details of sample selection are in Chapter III.

Organization of the Study

A brief discription of the salient features of the remaining chapters is given below.

Chapter II traces the development of the empirical literature relating to the characteristics of return distributions and the effects of marketability.

Chapter III details the design of the experiment and the collection of data. The constraints placed on the sample are examined in depth. The properties of discriminant analysis are discussed with particular emphasis being placed on its applicability to the present study.

Chapter IV reports the results of the experiment developed in the previous chapter. These results are then subjected to a statistical analysis to determine their significance. Specific conclusions are then drawn relative to the analysis.

Chapter V summarizes and extends the conclusions arising from the empirical testing. Additionally, the chapter interprets the findings relative to their implications for investment decisions. Suggestions for further research are also provided.

Chapter II

LIQUIDITY AND THE CHARACTERISTICS OF RETURN DISTRIBUTIONS

Introduction

The study of the heterogeneous nature of market return distributions is of recent interest. One aspect has been the recognition that differences in marketability might result in systematic differences in the distributional characteristics of returns. This chapter reviews the relevant literature and develops a set of testable hypotheses. Many of these articles contain information suggesting a relationship between marketability and the characteristics of return distributions, although, in many cases, the article does not directly address the possibility.

The literature is grouped into three categories: distributional characteristics of market returns, effects of liquidity[1] on the market mechanism, and effects of liquidity on market return distributions. The first two provide a foundation for the third and allow the reader to develop an understanding of the research and theories developed to date.

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[1]Technically, marketability refers to the ability to buy and sell a security while liquidity also requires a stable underlying price structure. Throughout this dissertation the two will be used synonymously, however.

Distributional Characteristics

The study of return distributions lacked rigor until the development of the Sharpe-Lintner capital asset pricing model. Earlier research[2] centered on the existence of randomness in successive price changes. Where distributional characteristics were examined, the results consistently showed distributions sufficiently "close" to normal to lead most people to believe market generated returns were normally distributed. With the development and acceptance of the capital asset pricing model, critical attention was given to the nature of market generated return distributions. This attention was an outgrowth of the interest in the empirical use of this model, requiring estimation by ordinary least squares regression and its attendant assumption of normality in the observations. If the normality assumption is violated, the regression model and individual estimated coefficients cannot be tested for significance. Deviations from strict normality were found and formed a systematic pattern: the same type of departures showed up in each study. This led researchers to attempt to identify a statistically recognizable distribution which would provide a better approximation of the empirical results than the normal. The question of an appropriate distribution has yet to be answered, but these studies have provided a number of insights into the important characteristics of market generated return distributions.

Three major alternatives have been considered in the literature: a stable Paretian distribution, a compound events distribution, and a

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[2]Various early studies are reported in Paul Cootner, ed., _The Random Character of Stock Market Prices_ (Cambridge: MIT, 1964).

scaled Student t. The following three sections examine the literature on each.

Stable Paretian Distribution

Even before the development of the capital asset pricing model Mandelbrot[3] questioned the normality of returns distributions. He noted that distributions of daily and monthly cotton price changes consistently had the property of leptokurtosis, that is, the distributions were more peaked and had higher probabilities in the extreme tails than a normal distribution. He proposed that a better representation of the distributions could be found in the stable Paretian family.[4] Mandelbrot considered only symmetric members of the family, as his empirical distributions did appear symetric even under close scrutiny. Within the symetric class only one parameter is needed to identify the form of the distribution, the characteristic exponent. The other parameters of the distribution shift its location and alter the scale,[5] but do not change the basic characteristics.

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[3] Benoit Mandelbrot, "The Variation of Certain Speculative Prices," _Journal of Business_, 36:394-419, October 1963.

[4] A brief discussion of the properties of the family of Stable Paretian distributions can be found in Appendix A.

[5] The terms location and scale will be used several times during the discussion of distributional properties. Both are generalized terms so as to be compatible with all of the distributions that will be discussed. A location parameter is comparable to the mean in that it determines the position of the distribution along the number line without affecting the shape. It does not, however, necessarily have the other properties of a mean. The scale parameter has a similar relationship to the standard deviation. The scale parameter determines the dispersion exhibited by the distribution. It may not have all of the other properties normally associated with a standard deviation.

When Mandelbrot set the parameters so as to provide the best fit to the empirical data, the resulting distribution had the properties of a well defined mean and the leptokurtosis that he and the early researchers had observed, but lacked defined moments of an order higher than the mean. This created the problem of a lack of statistical tools available for use when the higher moments do not exist. Mandelbrot published a supplementary article in 1967,[6] applying the stable Paretian distribution to several other series of speculative price changes. These results were comparable to those obtained in the first paper.

After developing improved methods of parameter estimation, Fama[7] also applied the stable Paretian distribution to sepculative price series, in particular to daily and monthly price changes. He, too, found the best set of parameters led to a distribution with a finite mean, symmetry, leptokurtosis and no defined higher moments. Fama's work did not lead to a complete confirmation of the stable Paretian hypothesis. Instead he found a lack of stability. A convergence of the normal distribution seemed to result as the differencing interval used in calculating price changes increased from a day to a month. The tails seemed more consistent with a normal distribution and the parameters approached those which would be associated with a normal distribution,

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[6]Benoit Mandelbrot, "The Variation of Some Other Speculative Prices," Journal of Business, 40:393-413, October 1967.

[7]Fama's work originally appeared in the form of numerous articles, some of which were done with coauthors. Fama subsequently authored a book which assembled the information into a unified body. For the concise version see Eugene F. Fama, Foundations of Finance, (New York: Basic Books, Inc., 1976), pp. 14-57.

in direct conflict with the stability assumption implicit in the definition of the stable Paretian distribution. Although not admitting that the distribution was in error as a description of the series, he did note that additional work would be necessary to establish the stable Paretian as the appropriate distribution.

Mandelbrot and Fama's research formed the basis for a number of papers dealing with refinements and verification of the use of the stable Paretian distribution as a description of the market generated return distribution. Grube and Dowell[8] looked at "clean" series of stock returns, data from time periods when no "material firm specific" information was being disseminated. They felt that firm specific information might disrupt the return generation process and thus the return distribution. The clean series, in their opinion, would give a clearer picture of the dominant distribution facing the investor at most points in time. Although their analysis of the empirical data suffered from some theoretical errors, their work indicated that returns did generally conform to the stable Paretian distribution family. In obtaining their "clean" series they were confined to working with 30 or less observations, which may have affected the results. The primary result of their efforts is that they showed that the distributions are insignificantly affected by identifiable firm specific information. Essentially they showed that these events generally generate only trivial distortions in the distribution of returns.

[8] R. Corwin Grube and C. Dwayne Dowell, "Common Stock Return Distributions During Homogeneous Activity Periods--An Extension," (paper presented at the meeting of the Southwestern Federation of Administrative Disciplines, Dallas, Texas, March, 1978).

Although accepting the idea of a stable Paretian distribution, Fielitz and Smith[9] found the idea of symmetry unacceptable. In their study of 200 daily return series, each almost five years in length, covering the mid to late sixties, they found considerable evidence of positive skewness. They stopped short of making any strong statements, since all of the series may have been affected by a common market function, which they did not account for. Fielitz continued this line of inquiry, generating more exact results[10] by analyzing the residuals of a market model. This allowed him to treat the 199 distributions as essentially independent samples.[11] The results of this study were in essential agreements with the results of the first, although the degree of positive skewness was less than previously found. In neither of the papers does Fielitz offer a theoretical reationale for the occurrence of skewness.

A number of other studies have been done examining empirical return distributions in relation to the stable Paretian hypothesis. Each of these have found irregularities which seem to be consistent across the samples which they used. Teichmoeller[12] examined a group of thirty stocks, including some preferred issues, estimating the characteristic

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[9] B. D. Fielitz and E. W. Smith, "Asymetric Stable Distributions of Stock Price Changes," Journal of the American Statistical Association, 67:813-814, Dec. 1972.

[10] Bruce D. Fielitz, "Further Results on Asymetric Stable Distributions of Stock Price Changes," Journal of Financial and Quantitative Analysis, 39-55, March 1976.

[11] The sample of stocks was for the same as the first study with the exception of the loss of the data for one stock.

[12] John Teichmoeller, "A Note on the Distribution of Stock Price Changes," Journal of the American Statistical Association, 66:282-284, June 1971.

exponent of the best-fit stable Paretian distribution for the daily price relatives of each of the thirty stocks. In addition exponents were estimated for distributions consisting of sums of consecutive daily returns. Three sets, sums of 2, 5, and 10, were constructed for each stock. The results were not entirely consistent with each other, but in a majority of instances there appears to be a slow convergence toward the normal distribution with larger sums. This is not to say that the normal is the limiting distribution, but simply that a drift in that direction appears. The convergence was far slower than what would have been expected had the distributions been the results of sampling from a group of normal distributions wtih differing parameters. The main result of this study was to question the stability, over addition, of the distribution.

The results of more comprehensive study, done by R. R. Officer,[13] paralleled those of Teichmoeller, as far as that research had gone. In an extension of the study of additive stability, Officer examine sums of fifteen and twenty daily returns and one to five month returns. A solid pattern of convergence toward the normal distribution (i.e., a thinning of the tails of the distributions) was very apparent through sums of up to twenty days. This convergence appeared to stop as the sums of returns of multiple months were considered, falling short of a normal distribution.

A second area examined in Officer's study was measurement of dispersion. Contrary to what would have been expected from a true stable Paretian distribution, the standard deviation was found to be a

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[13] R. R. Officer, "The Distribution of Stock Returns," Journal of the American Statistical Association, 67:807-812, December 1972.

well behaved measure of dispersion. This is highly suggestive of a distribution with a finite second moment, but not truly stable when dealing with returns based on short holding periods.

In an attempt to verify the results of the two previously mentioned studies, Barnea and Downes applied the same procedures to a random sample of 81 stocks.[14] They also found a strong indication of a lack of stability in the empirical distributions. In addition they found that the distributions varied considerably from stock to stock with respect to the extent of the leptokurtic tendencies. They stopped short of investigating the cause of this variability, offering simply that more work is needed.

A note published by Robert Hagerman[15] provided a broad based empirical study of the stable Paretian hypothesis. Of particular interest is the grouping applied to the data. Previous studies had been based on small samples based strictly on NYSE data. Hagerman used a large sample from the NYSE and the AMEX. Separate analyses were performed for each exchange. It is well acknowledged that some differences exist between the two markets. In particular, the listing requirements tend to separate the firms by size and strength with the smaller and weaker firms generally being relegated to the AMEX. It is not clear to what extent this separation exists or what effect it should have on market generated return distributions. The results of this study, in fact, show no significant difference in the distribution of returns.

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[14]Amir Barnea and David H. Downes, "A Reexamination of the Empirical Distribution of Stock Price Changes," Journal of the American Statistical Association, 68:348-350, June 1973.

[15]Robert L. Hagerman, "More Evidence on the Distribution of Security Returns," Journal of Finance, 33:1213-1220, September 1978.

This may have resulted from a lack of separation or a lack of breadth in the testing. The only test performed involved calculation of the characteristic exponent of a best fit stable Paretian distribution. The calculated exponents were then formed into a distribution for each exchange, with only summary statistics about this distribution presented. Visual inspection of these summary statistics indicate that the distributions of returns are not distinguishable from one another for the two exchanges.

Unfortunately, this testing leaves many unanswered questions. The methodology employed by no means provides proof of any hypothesis with regard to the similarity of return distributions from two populations with different characteristics. It is quite possible that there is a sufficient overlap in the characteristics so that any differences were small enough to be lost by the averaging process or by insufficient rigor in the testing.

Hagerman also examined the characteristic exponents of distributions generated from nonoverlapping sums of returns. For both exchanges, his results were in agreement with most authors. The estimated characteristic exponents rose significantly as the sum size increased, but failed to reach the level necessary for a normal distribution.

The final work that will be examined with respect to the stable Paretian distribution was done by Hsu, Miller, and Wichern.[16] The major thrust of their work was to question the stability, studying a sequence of nonoverlapping sums of successive observations, is not a robust test

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[16] Der-Ann Hsu, Robert B. Miller, and Dean W. Wichern, "On the Stable Paretian Behavior of Stock-Market Prices," Journal of the American Statistical Association, 69:108-113, March 1974.

against some types of nonstationarity. Of particular interest to the study of return distributions is their finding that this test is not sensitive to irregular and relatively infrequent shifts in the scale parameter. They suggested, and tested, a modification to the test which proves highly sensitive to these changes. The approach used is to apply a randomization process to the sequence of returns before testing. Their tests on four highly liquid securities indicate that, in fact, there is instability in the scale parameter of the type which was previously undetectable. This work unfortunately, leaves more questions than it answers. Although they show that instability in the scale parameter exists, they fail to demonstrate that the underlying distributions are normal. In particular their results are such that a Gaussian hypothesis is highly suspect. As was observed in several other works, the characteristic exponent failed to converge to the level necessary for a normal distribution. They also made no attempt to determine the average frequency of the apparent shifts in the scale parameter. Mention is made of the availability of a test for such shifts, but it is not applied. It appears that they at least suspect that these shifts in the dispersion parameter were fairly frequent since it would have been very easy to determine the underlying distribution if there existed lengthy homogeneous periods.

Analysis of the Stable Paretian Distribution

Several observations consistantly appear in the works just reviewed. The most common is that the observed market return distributions were not stable under addition, indicating that the holding period is a significant factor in the observed distribution's characteristics. None of the researchers investigated the cause of this phenomenon.

Although several explanations are possible, one is of particular interest: as the effective holding period is increased the number of transactions during the holding period are also increased. This is quite similar to the volume relationship between low and high marketability stocks. As such, a hypothesis that leptokurtosis should decline as marketability increases would seem in order.

Three additional points found extensive support. First, strong support was found for assuming that the second moment of the empirical distributions is a well behaved measure of dispersion. Second, positive skewness was observed in many cases. Third, the distributional properties varied considerably from stock to stock. None of the studies reviewed examined the cause of these differences. Again, although many explanations are possible, one source of the observed differences could easily be marketability.

Compound Events Distribution

The compound events distribution is, by far, the most complex approach that has been taken in the attempt to find a distribution that closely resembles the empirical distributions that have been observed. In this context, a distribution is developed based on a set of distributions, each of which is assumed to be the underlying distribution for a portion of the observed data. The specific distribution underlying a given observation is based on an additional random selection process. A theoretical justification for such an approach can be developed very easily on the basis of information flows.[17] Not all information coming

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[17] S. James Press, "A Compound Events Model for Security Prices," Journal of Business, 40:317-335, July 1967.

to the market is of equal significance. This could result in a set of return generating distributions, each associated with a specific level of informational significance. If it is assumed that the flow of information to the market is a random process itself, the result of the system is a compound events distribution.

S. James Press[18] is the only author to attempt to develop such a model. Others have mentioned such a model as a possible alternative, but have not pursued the matter.[19] Press, presenting a very rigorous theoretical development as well as a well-developed estimation procedure, considered a model based on two normal distributions and a Poisson selection process. In moving from the general case to the estimation of the model for specific return series, many problems were encountered. Press attempted to fit the model to monthly data for the period 1926 to 1960 for only ten of the stocks used in the Dow Jones Industrial Average. The very small sample size is not explained in the article. In many instances parameter estimation turned out far less accurate than anticipated. In particular, the estimates seemed to be highly dependent on the first observation and the last observation in the series.

In several instances Press was able effectively to model the observed price changes, supporting the nonstationary variance hypothesis mentioned by Hsu et al.[20] Specifically, the results indicate that

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[18] Ibid.

[19] Problems encountered by Press in the estimation of the parameters of the model seem to have discouraged further development of this approach.

[20] Hsu, op. cit.

dispersion shifts are quite common and random in direction. Therefore, the distribution facing the typical investor is not the distribution which generates the individual returns, but is a combination of such distributions where the combination is fairly stationary over time. In this situation the apparent distribution would not be stable, but instead would approach some limiting distribution of a type similar to the underlying process.

Analysis of the Compound Events Distribution

Two relevant conclusions can be drawn from the work on a compound events distribution. It is demonstrated, again, that the holding period is a relevant parameter for the determination of the characteristics of market generated return distributions.

The second important conclusion is the finding of instability in the variance. In those cases where parameter estimation appeared to be fairly accurate, the number of changes in the variance was quite high. Press did not pursue the cause of these changes. If his theoretical justification has any merit the potential for differences in instability, as a result of differences in marketability, is quite great. Press developed his theory on an assumption that the type of information flows determined the underlying distribution at a given point in time. It is very possible that the structure of information flows are closely related to marketability. High levels of marketability may correspond to high investor interest levels. This high level of interest should produce more efficient information flows. The differences in efficiency may generate different variance instability characteristics, which would produce return distributions that exhibit non-similar forms.

Scaled Student t Distribution

A direct response to the possiblity of an unstable dispersion parameter in the distribution of returns is the application of a sampling distribution. The most promising is the Student distribution. It is symmetric, has the characteristic fat tails, and converges to the normal in an orderly manner as the degrees of freedom parameter increases. However, it is flatter than the normal in the area about the mean, which does not coincide with the empirical distributions of stock returns. An approach is available to overcome the problem of the flattened centroid. If the distribution is standardized by dividing by its standard deviation as opposed to its scale parameter, the results exhibit peaked centers while maintaining the other desired properties.[21]

The first extensive application of the scaled Student t distribution was done by Praetz,[22] a rigorous model development building directly on the now famous work of Osborne.[23] The essence of the argument is that Osborne's assumption that the variance of the returns is constant was erroneous. Praetz postulates that changing expectations of investors will alter the variance of returns. On the basis of this hypothesis he assigns a gamma distribution to the various parameters of Osborne's model. The resulting distribution has the form of a Student distribution with a scale factor of the type mentioned above. It is

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[21] A more complete explanation of this scaling procedure can be found in Appendix B.

[22] Peter D. Praetz, "The Distribution of Share Price Changes," Journal of Business, 45:49-55, January 1972.

[23] M. F. M. Osborne, "Brownian Motion in the Stock Market," Operations Research, 7:145-73, March-April 1959.

obvious that the choice of a distribution for the variance will affect the shape of the resulting distribution. Praetz offers little in the way of justification for his selection other than it is well defined, is unimodal, and has the necessary characteristic of being strictly non-negative.

The empirical study included in his paper was based on weekly observations for seventeen share-price indexes. The data was collected from the Sydney Stock Exchange and covered a nine year period. His goal was to show that the scaled Student t distribution was superior to the compound event and stable Paretian distributions. For each of the alternative distributions, the parameters were estimated so as to provide the best fit possible to the actual data. A Chi-square test was then applied to each relative to the actual data. For each of the seventeen series examined the scaled t provided the best fit. These results provide considerable support for the idea that the distribution of returns is unstable over addition (holding period).

A second result of interest is found in the "degrees of freedom" parameter of the best fit distribution. In fifteen of the seventeen series the parameter was found to be four or larger, indicating that the first four moments of the distribution are defined and finite. This is in direct conflict with the stable Paretian distribution for which all moments past the first are undefined, but in agreement with several of the previously discussed articles which found strong evidence that at least the second moment is defined.

Additional work in this area was subsequently done by Blattberg and Gonedes.[24] The development of a scaled t distribution in their work is far more empirically motivated. Their goal was imply to validate Praetz's findings and examine the implications of the scaled t distribution on investment theory.

The empirical testing procedures employed relied on a completely different approach. Only two alternative distributions were considered, the scaled Student t and the stable Paretian. The sample consisted of the thirty securities used in the Dow-Jones Industrial Average, measured on a daily basis.

The specific method of analysis also deviates significantly from what was used by Praetz. Blattberg and Gonedes applied likelihood ratios to the best-fit models. For every security tested, the scaled Student t distribution provided a better fit to the empirical data. Other tests based on nonoverlapping sums of successive returns also supported the superiority of the scaled Student t distribution, as a convergence to normality was generally present in larger sum sizes.

An examination of the "degrees of freedom" parameter that was derived for each of the securities, again, confirms the strong likelihood that the first four moments of the return distributions are defined and finite.

Analysis of the Scaled Student t Distributions

The scaled Student t distribution has proven to be the most effective distribution that has been tested for describing the market

[24] Robert C. Blattberg and Nicholas J. Gonedes, "A Comparison of the Stable and Student Distributions as Statistical Models for Stock Prices," Journal of Business, 47:244-280, April 1974.

generated return distributions. An examination of the parameters of the best-fit distribution for each stock or index that was evaluated revealed two interesting properties. In almost all cases the parameters are such that at least the first four moments are defined and finite. Also there is a great deal of variability in the parameters from one stock to another. Although the variability has been noted by each author no effort was put forth to explain its cause.

Summary of Distributional Properties

On the basis of the foregoing discussions, several points should be made about the properties of return distributions. It is very obvious that there is still a great deal of disagreement about the distributions underlying market returns. Even so, several items continually reappear in the literature. It is generally accepted that the distributions are unimodal and exhibit leptokurtosis. A slight amount of positive skewness was found by several authors, although generally felt to be insignificant. With regard to stability, the literature is mixed. A significant body of literature does exist to suggest, at least, that in many instances the distribution of returns is not stable, but converges toward a normal distribution as the holding period is increased. There is also considerable evidence that this convergence falls short of reaching a normal distribution as its limit.

The existence of finite moments is strongly supported for the mean and variance. Some support has also been found for the existence of well defined third and fourth moments. The existing research in this area has been focused on the impact of the alternative distributions on existing investment theory and thus have primarily centered on the first two moments.

The existing research also supports a hypothesis that the distributional form differs between stocks. These differences appear to extend well beyond the mean and variance captured by the capital asset pricing model. Press and Praetz are the only authors to specifically recognize this fact and offer an explicit incorporation of the concept in their model. The results of their testing and that done by most other researchers showed these differences, but the differences were not afforded an explanation. None of the work in this area has attempted to isolate factors which are causing these differences.[25] It would appear that they are caused by one or more characteristics of the specific firm. It is not as clear that these differences are eliminated through normal diversification methods. If not eliminated, the true risk of the portfolio could be significantly greater than necessary.

Liquidity and Market Returns

Up to this point the examination has dealt strictly with establishing the characteristics of return distributions. No evidence has been presented to justify the stock to stock differences that have been found. One possible source of these differences is liquidity. The available research is very indirect, with the primary thrust in the Over-the-Counter (OTC) market. Before the research investigating the OTC market can be combined with the previously discussed works, it is necessary to demonstrate that the market places are compatable in terms

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[25] Current empirical work on the application of arbitrage pricing theory is beginning to address the question, but it has not yet advanced to the point where definite conclusions can be drawn. For a more complete discussion of this theory see Richard Roll and Stephen Ross, "An Empirical Investigation of the Arbitrage Pricing Theory," Journal of Finance, 35:1073-1103, December 1980.

of the return generation process. Prior to the development of the NASDAQ system,[26] such a relationship could not have been established because of the major differences in the market structures. Today, with the help of NASDAQ the speed of information transfer has increased to the point that it is reasonable to think that successive price changes are independent and follow a random walk. When these two conditions are present, it is said that the market is efficient in the weak form. If weak form market efficiency can be established in the OTC market, the results of the liquidity studies will be able to be combined with the previous results.

Basu and Witcher[27] tested the OTC market for the existence of weak form market efficiency comparable to that found in the New York Stock Exchange (NYSE). The empirical work undertaken was based on daily closing values of the NASDAQ composite index from February, 1971 through October, 1975. Their testing centered around the extent of serial correlation. A lack of significant serial correlation in a series is an indication of independence. Their results were quite comparable to those of similar tests conducted by Fama[28] on NYSE data. Significant

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[26]NASDAQ stands for National Association of Security Dealers Automated Quotations and consists of a computer network and the associated programs. The system provides up to the minute precise quotations to all security brokers and dealers that are members of the association. Additionally, large amounts of information are now available in the form of summaries and indexes that have been derived from the data in the system.

[27]Sambhu N. Basu and Alan H. Witcher, "Over the Counter Market and Market Efficiency," Unpublished working paper. Graduate School of Business Administration, University of Southern California, 1977.

[28]Fama, op. cit., pp. 14-57.

correlation was found for a lag of one day, while longer lags resulted in insignificant values. This indicates less than perfect weak form efficiency, but the deviation is not so great that it should induce any severe problems in evaluating the results of other studies of the OTC market and extending these results to the major exchanges.

The pricing of liquidity services, which is the determination of bid-ask spreads, has been extensively studied. Dealers and specialists are highly skilled investors with significant market power. It can be assumed that those factors that effect their pricing decisions also effect the return generation processes and in the end the market generated return distribution.

Tinic[29] examined the behavior of the specialists on the NYSE using several multiple regression models to determine the causes of the size and variability of the bid-ask spread. Market liquidity was defined and measured as average daily trading volume. The results of the model examining the size of bid-ask spreads indicated that average daily volume and bid-ask spreads are negatively correlated. The average daily volume was not significantly correlated to the variability of the bid-ask spread.

This line of inquiry was continued by Tinic and West.[30] They investigated the same questions as the earlier study, only this time the data was drawn from the OTC market and the Toronto Stock Exchange (TSE). Note the nature of the environment of the dealers under study: the NYSE

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[29] Seha M. Tinic, "The Economics of Liquidity Services," Quarterly Journal of Economics, 86:79-93, February 1972.

[30] Seha M. Tinic and Richard R. West, "Marketability of Common Stocks in Canada and the U.S.A.: A Comparison of Agent Versus Dealer Dominated Markets," Journal of Finance, 29:724-746, July 1974.

specialist, as examined by Tinic earlier, operates in a highly regulated monopoly, while the OTC dealer is subject to virtually unlimited competition. The dealer on the TSE is in a monopoly position and subject to only very loose control. It is easily seen that the actions of these groups are potentially very different. Both the TSE and OTC dealers are far less restricted in their activities than their NYSE counterparts. A study of their behavior should provide a somewhat clearer picture of what dealers consider to be significant variables in their decision making.

The models developed for both the OTC and the Toronto Exchange showed again that market liquidity was a prime factor in establishing bid-ask spreads, demonstrating that the previous findings were not caused by the particular environment of the NYSE specialist.

Ying[31] examined the relationship between marketability and return characteristics from both a static and a dynamic standpoint. Specifically, he attempted to relate the logarithm of daily price changes to trading volume and changes in trading volume. Data for this study consisted of daily closing prices from Standard and Poor's 500 Composite Index and daily volume on the New York Stock Exchange. The approach involved a series of analysis of variance tests to examine cross-sectional relationships. The results revealed that there exists a relationship between price changes and both volume and volume changes. The specific relationships found, however, do not lend themselves to generalizations.

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[31]Charles C. Ying, "Stock Market Prices and Volumes of Sales," Econometrica, 34:676-685, July 1966.

In an attempt to expand the study, Ying applied cross-spectral analysis to the data to detect any lag structures. This resulted in the finding of a four-day lag between several series. In general volume changes tended to lead price changes.

Analysis of Liquidity and Market Returns

Each of the studies that examined the relationship between liquidty and market returns reached an affirmative conclusion. It would appear, from the results of these studies, that liquidity does affect the market return generation process. The evidence presented does not, however, indicate in what way the resulting return distributions would be affected.

Marketability and Market Return Distributions

Only a handful of studies have been published dealing directly with the relationship between marketability and return distributions. The majority of the work has been done under the direction of Cohen as part of a major study of the return generation process. Toward this end Cohen, Maier, et al.[32] sought a theoretical explanation of the relationship. They choose the market value of shares outstanding for a security as the proxy for measuring marketability.

Since the supply curve is essentially fixed in the short run, they focused their attention on demand curve shifts. Two distinct causes of demand curve shifts were investigated. The first is new information

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[32] Kalman J. Cohen, Steven F. Maier, Robert A. Schwartz, and David K. Whitcomb, "The Returns Generation Process, Returns Variance, and the Effect of Thinness in Securities Markets," Journal of Finance, 33:149-167, March 1978.

which is available to all investors. This type of shift will not necessarily generate a transaction, but will cause a new quoted price. The second type of shift is referred to as "idiosyncratic" shift. It is the result of a change in the demand of an individual investor. The most common causes of this are a change in the individual's funds position, risk-return preferences, or reevaluation of the "value" of the security. When a large idiocynscratic demand shift occurs, they argue it will not only trigger a transaction, but more importantly that transaction will be at a price away from the existing equilibrium, thus creating a new equilibrium at that price. The extent of the price change that will result from such a transaction is dependent on the elasticity of demand in the vicinity of equilibrium, the prevailing price, and the percentage of the outstanding shares which are involved. When restated in terms of the return which will be generated only the elasticity of demand and the percentage of the outstanding shares being traded are involved in the determination of the demand shift.

Both types of demand shifts are assumed by Cohen, Maier et al. to be generated by mutually independent compound Poisson processes. Specifically, the occurrance of a demand shift is assumed to follow a Poisson process. The direction of the demand shift is determined by a Bernoulli random variable.

The analysis of the implications of the foregoing assumptions was undertaken on the basis of two sets of additional assumptions about investor behavior. In one case homogeneous expectations and separation dominate. In this situation it was assumed that transaction size is proportional to the total market value for that security. That is, the firms with the largest market value will have the largest transactions while smaller valued firms will be subject to proportionately smaller transactions. The study showed very rigorously that both the mean and variance of the resultant return distributions will be essentially independent of thinness.

If, instead, the trader is assumed to be acting under heterogeneous expectations and nonseparation, the implications are quite different. In this case, transaction size is considered to be constant and, therefore, independent of thinness. Here again, the expected return is shown to be effectively independent of thinness. But, unlike the previous case, variance is shown to be directly related to thinness. The variance is found to be larger for thinner issues, all else constant.

Further analysis of the equation for variance under conditions of nonseparation indicates that an active market-maker can reduce or eliminate the effect. Indeed, the role of the specialist in stock exchange is to make a market for his stocks, buying or selling for his own account to assure a continuous auction market and thereby provide liquidity and stability to the market. In negotiated markets, that is, in the absence of the specialist, there may be no market-maker active enough to provide sufficient liquidity to the market for every security. Testing of this hypothesis has not been done, leaving it strictly an hypothesis.

To summarize, their theory suggests that the expected return is independent of thinness regardless of the assumptions about investor behavior. The relationship between thinness and variance, on the other hand, is dependent on the relationship between thinness and transaction size. If it is strictly proportional the return variance will be unaffected by thinness. If relative transaction size is larger for thinner issues, variance will be directly related to thinness. It is also apparent that if the market-maker has and applies significant market power, he can alter this relationship. The results of the market-maker's activities are not totally clear at this point, since he has great latitude in his actions.

In another work, Cohen, Ness, et al.[33] empirically investigated some of the above tenets. Four stratified random samples were drawn based on total market value of a firm's common stock. Three of the sample were composed of 50 common stocks each from the NYSE, the American Stock Exchange (AMEX), and the Tokoyo Stock Exchange (TKYO). The fourth sample consisted of 28 stocks from the Rio De Janeiro Stock Exchange (RIO). This data was then subjected to regression analysis in an attempt to isolate the causes of return variance. The dependent variable was the variance of daily returns for a three month period. Three independent variables were included, two for the purpose of investigating thinness differences and one to measure information differences. The floating supply (FS)[34] and average price (P) were included to measure thinness while the turnover ratio (TOR)[35] was included to

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[33]Kalman J. Cohen, Walter L. Ness, Jr., Hitoshi Okuda, Robert A. Schwartz, and David K. Whitecomb, "The Determinants of Common Stock Returns Volatility: An International Comparison," Journal of Finance, 31:733-740, May 1976. It should be noted that this paper was completed prior to the theoretical paper of Cohen, Maier, et al., and as such does not constitute a proof of the hypotheses previously presented. It is discussed after the theoretical paper for the purpose of clarity.

[34]Floating supply is defined to be the number of shares outstanding less insider holdings.

[35]The turnover ratio is the number of shares traded in a month divided by the floating supply.

account for information differences. Dummies were also included to investigate each market separately. All of the variables were then converted to logs to eliminate scale disparity problems.

The results of the regression analysis were somewhat ambiguous. For all but RIO, price was inversely related to variance, supportive of the Cohen, Maier hypothesis. Further investigation revealed that many of the stocks on the RIO exchange had been manipulated during the interval covered by the study and thus did not generate normal return patterns. The coefficient of FS was negative for TKYO, but was insignificant for the two specialist exchanges. The reason for this is not totally clear, but two likely explanations are available: either specialists are effective, with respect to this variable, in eliminating the impact of thinness, or multicolinearity is distorting the relationships. Insufficient information is presented to determine which of these is the dominant factor.

The coefficients of TOR vary drastically in magnitude across the different exchanges. In all cases except RIO the sign if positive. This indicates that in general an increase in information flow increases variability, consistent with most theories.

In a review of the Cohen, Ness, et al. paper, Lessard raised several questions and criticisms.[36] Lessard noted that the variables chosen to measure thinness were not the best that were available. The product of the two used (i.e., FS*P) would have measured the value of shares outstanding. A second alternative that was suggested was the value of shares traded (FS*P*TOR). By separating FS and P the authors

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[36] Donald R. Lessard, "Discussion," Journal of Finance, 31:751-752, May 1976.

weakened their argument with respect to validating their theory and also made the analysis of their results much more difficult. This is due to neither variable by itself being clearly a measure of thinness.

Lessard also criticizes the variable used to measure information. It is his feeling that TOR probably contains some of the properties of thinness. This, too, would serve to cloud the analysis. Although alternative methods of controlling for information are available, it is not clear that any could be used effectively.

Senchack and Barnett[37] undertook a study of regional OTC securities. One segment of their work involved calculation of the average rates of return, standard deviation, and Fisher's skewness coefficient[38] for weekly return distributions for each of four indexes. Three indexes were the NYSE composite, the AMEX composite, and the NASDAQ composite. The fourth index was generated as an equally-weighted index of forty-seven OTC stocks of firms geographically based in southwestern states. One of the biggest differences between these indices is the thinness of the trading market involved.

An examination of the rates of return showed that the three national indices all exhibited average weekly returns near zero. The highest was the NYSE at .04 percent, while the lowest was the AMEX at -.04 percent. This represents approximately 2.1 percent per annum respectively. The submarket, however, exhibited an average weekly return of nearly three tenths of a percent (seventeen percent annually).

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[37]Andrew J. Senchack, Jr., and William S. Barnett, "Price Behavior in a Regional Over-the-Counter Securities Market," Unpublished working paper, March 1977.

[38]Fisher's skewness coefficient is the third moment of a distribution divided by the cube of the standard deviation.

A closer examination of the returns of the individual securities in the submarket showed that three of these securities had weekly returns averaging in excess of one and one-half percent (117 percent annually), while the lowest was -.33 percent. Although this may have biased the returns in the submarket to some extent, it is unlikely that this bias accounts for all of the differences that were found.

When the standard deviation was examined it was found that it increased with thinness with the exception of the AMEX which actually had a smaller standard deviation that the NYSE. If there is not a significant degree of difference in the inherent risks of stocks which make up the different indexes, the results of the study of standard deviations can be used to examine the behavior of investors relative to the separation hypothesis. If total separation exists the standard deviation should be the same for all indices. On the other hand, if no separation exists, each stock would be subject to larger independent price movements. When put into an index, however, these independent movements should essentially cancel out, again leaving the standard deviation of the indexes the same. Neither of these are supported by Senchack's results. A third situation is supported whereby investor behavior lies somewhere between these two extremes, resulting either from a market made up of investors that belong in each of the two groups, or investors who perceive stocks to belong to groups, such as industries, cyclicals, etc., and follow separation within the groups and nonseparation between groups. In either, the cancelling process would not completely eliminate the increased volatility of the thinner issues, thus producing the pattern that was observed.

The third distributional measure employed by these authors was Fisher's skewness coefficient. A figure of .017 was reported for the NYSE, indicating a very slight amount of positive skewness, and is comparable to the results obtained by Blume[39] and Friend and Blume.[40] The AMEX showed slightly more skewness, .072, while the NASDAQ index reflected much more skewness, .271. The regional submarket which was substantially thinner than the others had a skewness coefficient of .637. As mentioned earlier the submarket contains three outliers having extremely high positive returns. If these are abnormal stocks, the skewness coefficients would be upwardly biased. It is unlikely, however, that even the elimination of these three would lower the skewness coefficient sufficiently to alter the validity of the apparent relationship between thinness and skewness.

Senchack and Barnett also undertook a comparison of each of the empirical distributions to the normal using goodness of fit tests and a descriptive analysis of the cumulative distributions. In all the cases the empirical distributions were significantly different from the normal. In addition to the volatility and skewness properties that have already been discussed, evidence of leptokurtosis was also found. The degree of leptokurtosis appeared to increase as thinness increased. The descriptive analysis employed to arrive at this conclusion was not regorous, but was adequate to suggest the relationship.

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[39] Marshall E. Blume, "Portfolio Theory: A Step Toward Its Practical Application," Journal of Business, 43:152-173, April 1970.

[40] Irwin Friend and Marshall E. Blume, "Measurement of Portfolio Performance Under Uncertainty," American Economic Review, 60:561-575, September 1970.

Analysis of Marketability and Market Return Distributions

It has been shown both theoretically and empirically that marketability and the market generated return distributions are related. Cohen, Maier et al. showed that, under realistic assumptions, the variance should be related to marketability. They also demonstrated that the mean return should be independent of marketability.

The two empirical studies that have been done indicate that the characteristics of the return distributions appear to be related to marketability. It is, however, necessary to keep in mind how marketability differences were achieved in these two studies. In both cases marketability was determined on the basis of the market place where the issue was traded. As such, at least some of the results may be attributable to differences in market structures.

Conclusion

Throughout this chapter we have examined the contributions of others that have been aimed at the relationship between marketability and distribution of returns. Much of this discussion has involved works dealing with related topics.

The existing body of research in the area of distributional properties reflects a great deal of controversy. Although many researchers have investigated the properties of return distributions there is little agreement in their results. Most researchers agree that there is some degree of leptokurtosis. In particular, overly thick tails are found in almost all cases. The question of symmetry is not as well resolved. Although many researchers found essentially symmetrical distributions, others found tendencies for positive skewness. This disagreement indicates that skewness should not be ignored in any study dealing with return distributions.

The differences found between studies and the differences from one stock to the next found within each study, strongly support a contention that the distribution underlying the return generation process is not homogeneous across all stocks.

Research evaluating the effect of thinness on the trading market for a security has centered on the OTC market or on market makers. In each of these studies, thinness is shown to be a significant contributer to stock-to-stock differences. None of these studies deals with the return distributions. They do support study of the relationship of marketability and return distributions since, on the basis of those studies, thinness does influence the actions of the well trained market makers which in turn, could be expected to influence the return pattern.

Very little work dealt specifically with the relationship of marketability and return distributions. The one theoretical examination showed that under most assumptions of investor behavior, marketability will induce an increase in variance but not influence the mean return. Under the assumptions of homogeneous expectations and separation, however, marketability would not influence the return distribution. That investigation did not pursue the relationship beyond the second moment of the distribution and, as such, left many questions unaddressed.

Empirical examinations of possible effects of marketability on return distributions are very limited. Those studies do generally support such a relationship, but have been very limited in scope and have had methodological irregularities. The most significant problem in the past studies has been the use of indexes. The present study will employ a different approach which does not require the use of indexes. It is hoped that this will clarify the extent and nature of the relationship.

The next chapter describes the approach that is to be used for this investigation, including the procedures for data screening and the testing procedures that will be employed.

Chapter III

RESEARCH DESIGN

This chapter describes the construction of the sample and the techniques used in analyzing the data. The first section of this chapter details the construction of the basic sample and a portfolio sample. Included in that section are a consideration of the limitations and restrictions that were imposed and a definition of the measure of marketability that is employed. The second section discusses the use of discriminate analysis to provide insights into the effects of marketability. The use of graphical analysis, as a supplemental technique is also discussed. Finally, as part of the summary a series of questions reflecting the possible effects of marketability are presented. These questions are answered in the next chapter.

Sample Design

The choice of the sample was based upon several factors: available data sources, sample period, exchange listing, measure of marketability to be used, grouping requirements, holding period, and return calculation procedure. The basis for each decision will be discussed in the remainder of this section.

Any empirical analysis of return distributions requires massive amounts of data. Each company must be represented by a continuous stream of prices that is long enough to allow the approximation of distributional parameters. Also the sample must contain a sufficient number of companies to minimize the significance of random factors and undesirable mathematical properties associated with the techniques used. Because the analysis will be nonparametric in nature, absolute minimums for the number of companies to include cannot be determined. It is obvious, however, that the data requirements are very large. Time and expense constraints necessitate the use of data already in machine usable form. Three such data bases were available. The first is the Center for Security Price Research (CRSP) file containing daily returns for all stocks listed on the New York Stock Exchange and the American Stock Exchange. The other two are Compustat files, the monthly Price-Dividends-Earnings (P-D-E) file and the Annual Industrial File. The coverage of the Compustat files is less complete than that of CRSP. In order to utilize a consistant sample throughout, the available data on the available source files were compared and only those firms with extensive data available on all three were considered for inclusion in the sample.

The proper holding period length to study is open to considerable question. To some extent the decision is limited by the availability of data. The shortest period for which data is available is one trading day. The use of a short holding period is supported by several very valid arguments. Short holding periods should closely reflect the return generating distribution whereas a return distribution based longer holding periods is the sum of many distributions from shorter holding periods. This summation process can mask characteristics through a process akin to the Central Limit Theorem. Additionally, the use of longer holding periods requires the collection of data over a proportionally longer period of time in order to obtain a sufficiently large number of returns to accurately approximate the return distribution. This can cause a stationarity problem because of the dynamics of the economy and the company under study.

Arguments are also available to support the use of a holding period longer than a day, the most important of which is the lack of short term speculation or other trading in illiquid securities. If short term interest is low the distribution of daily returns may be nonsensical. If the majority of potential investors are basing decisions on longer holding periods the daily distribution could be nothing more than a mathematical curiosity.

Since there are contradicting arguments as to an appropriate holding period, two will be used, daily and monthly. This may provide some information about the validity of each of the arguments and will allow the determination of the effect the holding period has on the generality of other results. A period longer than one month is not considered since data would have to be gathered over many years in order to assure a large enough sample to generate an accurate sample distribution.

The selection of a sample period from which to generate the distributions relied on many of the same considerations. For the daily returns distributions, an arbitrary choice of one year beginning on January 2, 1978, was made, resulting in 252 returns for each company.

With monthly returns the trade-off between accuracy and stability is much more important. Fama[1] suggests an interval of five to seven years. It is his feeling that a longer sample period provides too great

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[1] Eugene F. Fama, Foundations of Finance (New York: Basic Books Inc., 1976).

a chance that the firm or the world will change significantly, altering the return generating process and thereby producing inaccurate estimates of the underlying distributions. Therefore, for monthly return distributions an interval of five years was chosen beginning July 1974 and ending June 1979.

In order to avoid the possibility that observed differences in return distributions could be caused by structural differences in the trading market, it was necessary to restrict the sample to stocks traded in comparable environments. Hagermann[2] found no significant difference in the observed return distributions when comparing the New York and American Stock Exchanges. On the basis of these results, the available data from the two exchanges was combined. Stocks which are traded over-the-counter were excluded on the basis of the obvious differences in the trading mechanisms. On an absolute basis, this eliminated all of the truly illiquid stocks, as they are traded OTC. It was not possible to replicate the study using only OTC securities. No OTC firms were in the CRSP daily returns data and insufficient numbers were available on the Compustat P-D-E files. Because of the differences in the trading mechanisms between the OTC market and the organized exchanges, it was not considered prudent to mix the two in the monthly returns sample. Therefore, only data from the organized exchanges is used and the testing centers around relative liquidity differences in that sample.

Each of the above mentioned criteria reduced the sample size. Eventually the common stocks of 1,470 firms met all of the requirements. This quantity of data indicates the restrictions that have been

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[2] Robert L. Hagerman, "More Guidance on the Distribution of Security Returns," Journal of Finance, 33:1213-1220, September 1978.

established do not confine us to only a very select group of stocks. Some bias may exist as the requirements for inclusion do favor the larger more established firms. However, the quantity available should provide significant breadth in the usable data.

Marketability Strata

Through the years many different measures of market thinness have been suggested. From an abstract standpoint, the measure should reflect the degree to which a reasonable block of stock can be traded in the market without a significant impact on the price. This definition is not a workable measure. The two most acceptable measures are average dollar trading volume and percent of outstanding shares traded. The second is the more commonly used. Average dollar trading volume contains a potential bias relative to firm size. Small firms tend to have small dollar trading volumes as compared to large firms. Small firms also tend to be newer and more risky. This could yield results that reflected risk differences instead of ones induced by thinness. The use of percent of outstanding shares traded avoids this bias and does not induce any other of significance. It was chosen for use here.

As mentioned previously, the determination of an appropriate sample size can not be accomplished without some knowledge of the nature of the effect of thinness. If thinness become a factor in the determination of returns only at the extremes, it would be necessary to estimate the upper and lower bounds where this occurred and draw samples from each category. Opposing this, if the effect of thinness is a continuous process, it would be necessary to select samples that had significant gaps between them in order to insure that similarities within each group exceeded those between the groups. A cluster analysis was done to determine if significant discontinuities could be found and used as group cutoffs. The analysis did not find any significant breaks in the data. This resulted in a decision to statify the sample and select groups from sèveral segements of the available data. A total of four groups were constructed, each containing one hundred firms. A group size of one hundred was chosen as the result of a trade-off between precision in estimation, a necessity for intergroup separation, and within group homogeneity. As in all statistical estimation the larger the sample the more accurate the estimates will be if the data is homogeneous. In this case, the assumption of homogeneity is suspect for very large sample sizes. The absence of significant breaks in marketability results in a need to divide the data arbitrarily, however. Relative homogeneity can be created by assuring that more heterogeneity exists between the groups than within. Group sizes of one hundred provide such a situation, while providing a large enough sample for all testing that is to be considered.

The total sample of 1,470 was first rank ordered by marketability for the year 1978. The two extremes were excluded to avoid possible bias caused by their proximity to the limits of obtainable marketability. One hundred securities were omitted at the lowest end. The next group of one hundred were used as the group to represent low marketability, with trading percentages ranging from 8.6 to 11.5. Three hundred were then skipped to provide a break between groups. The following one hundred were then used to represent the second group, being relatively in the lower central area. Trading percentages for this group ranged from 17.8 to 19.6. The third group of one hundred was positioned with a gap of two hundred and represents the upper central area, with trading percentages of 24.1 to 27.0. High marketability, with trading percentages of 41.1 to 51.7, comprised the fourth group of one hundred and was positioned with a gap of three hundred. The final 170 securities exhibiting the highest marketability were omitted.

Return Calculation

Returns were calculated with dividends included. The CRSP Daily Returns file is generated with dividends already accounted for. In an attempt to keep as much consistency as possible between the results for the two lengths of holding period, monthly returns were also calculated with dividends included. Additionally all returns were adjusted for any significant change in the capitalization of the firms in the sample. The natural log of the returns was chosen as the value to be used in testing. This number represents the continuously compounded return for the holding period, and it also corrects for the positive skewness observed in distributions of returns.

Summary

The sample used in this study contains daily and monthly return data for four hundred firms. The daily data covers a period of one year, while the monthly data covers five years. The sample contains four sub-samples of one hundred firms each, selected on the basis of the percentages of the outstanding shares that were traded each year.[3]

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[3] A list of all four hundred firms included in the sample is available in Appendix C.

Analyses

Initially return distributions are generated for each security and each holding period length. Five statistical measures will be derived to characterize each distribution: the mean, the standard deviation, Fisher's skewness coefficient, Fisher's kurtosis measure, and the studentized range. The first two are very well known and need no further explanation. The remaining three are less familiar. R. A. Fisher[4] pioneered the use of alternate measures for skewness and kurtosis. He divided the third and fourth moments by the third and fourth power of the standard deviation, respectivley. This results in a unitless relative measure in each case, and improves the ability to compare distributions with dissimilar standard derivations.[5]

The final measure is the studentized range. It is computed as the range divided by the standard deviation. Fama suggests this as an alternative to the fourth moment for measuring the extent of kurtosis.[6] The primary argument for its use is that it does not rely on the fourth moment which may not be defined in the theoretical distribution. The studentized range has gained sufficient acceptance to warrant its inclusion here.

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[4] R. A. Fisher, Statistical Methods for Research Workers, 14th Edition (New York: Hafner Publishing Co., 1973).

[5] A similar argument can be made for examining dispersion on the basis of the coefficient of variation instead of the standard deviation. This was not done, however, since it would eliminate the possibility of comparison to previous work. Also, significant distortions occur because the mean returns are so close to zero.

[6] Eugene F. Fama, Foundations of Finance (New York: Basic Books, 1976), 8-11.

The analysis of these measures is a considerable task. There are no statiscal tools that lend themselves directly to such usage. Multivariate analysis is dictated because of the multiple measures that are employed to characterize each distribution. The primary developments in multivariate statistics have been involved with parameteric tests requiring each measure to be drawn from a normal distribution. The statistical measures used in this study do not conform to this requirement, thus eliminating the possibility of full implementation of these techniques.

Primary Analysis

After examining the available techniques, discriminant analysis was selected for the primary analysis. The function of discriminant analysis is to find a set of functions that will best separate groups of data.[7] The technique was developed on the basis of normally distributed inputs, but will still yield unbiased regions of best separation when the normality assumption is violated, which is a sufficient condition for the analyses that will be used. Discriminate analysis involves the construction of a function for each group, representing the squared distance from that group centroid to a particular observation. That observation is then considered to belong to the group to which it is closest. The computational approach used to derive the functions is largely dependent on the equality or nonequality of the within-group covariance matrices and group sizes. In this study each group contains 100 firms, but the results of a pretest indicate that the within-group

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[7] C. R. Rao, Linear Statistical Inference and Its Applications (New York: John Wiley and Sons, Inc., 1965), 574-577.

convariance matrix differs from group to group. These conditions produce a generalized squared distance function from a point X to group t of:

$$D_t^2(X) = (X - \bar{X}_t)' \, S_t^{-1} \, (X - \bar{X}_t) + \ln |S|_t$$

where:

t is a subscript to distinguish the groups.

S_t is the covariance matrix for the observations within group t.

$|S|_t$ is the determinant of S_t.

X is a vector containing the variables of an observation.

$\bar{X}_t$ is the vector containing means of the variables in group t.[8]

Point X is then classified as belong to group u for that value of u = t that yields the smallest value of D_t^2 (X). If the groups are distinctly different the degree of correct classification of the points will be very high. The results of this classification process are generally presented in the form of a matrix. An example of such a matrix is shown in Table 1.

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[8] J. T. Helwig and K. A. Council, eds., SAS Users Guide, 1979 Edition (Raleigh, N.C.: SAS Institute, Inc., 1979), 83. The SAS software package was used for all discriminant analyses.

Table 1

Example of a Classification Matrix

	Classified into Group				
From Group	1	2	3	4	Total
1	8	1	1	0	10
2	1	6	2	1	10
3	1	2	7	0	10
4	1	1	2	6	10
Total	11	10	12	7	40
Percent	27.5	25.0	30.0	17.5	100.0

This example shows twenty-seven correctly classified observations out of forty which represents a correct classification percentage of 67.5, more than two and one half times as great as expected from random assignment. The probability is small that this large a difference would occur from groups that are in fact not different. The pattern of misclassifications is also important. If a large percentage of the classification ability is attributable to one group while the other groups have a large number of points misclassified amongst themselves, it would indicate that the one group was different from the others. The remaining groups would be considered nondistinct from one another.

Ideally, one set of data is used to create the discriminate function and a second used to determine the classification ability of the function. Because of the limitations on the availability of data, it was not possible to use a holdout sample. Preliminary testing indicated that this would not be a significant problem. In those tests the differences in classification ability between the classification sample and the post-test sample differed by no more than two percentage points, and appeared to be random in direction and magnitude.

The problem with using discriminant analysis with non-normal variable inputs lies in evaluating the results. The usual significance tests to evaluate the distinctness of the groups rely heavily on the normality assumption. As a result a new approach to analyze the results had to be developed. The only analysis available that does not rely on parametric testing is an examination of the classification ability of the model. If the regions as defined by the four levels of marketability are extremely different the probability of correct classification would be expected to be 1.0. If the four groups are actually drawn from one common distribution, the regions will represent one region and the classification would be a random event. This would result in a probability of correct classification of .25 (one divided by the number of groups).

A lower limit for statistical significance can be approximated by making use of the properties of our classification measure. Each observation is classified as either correct or incorrect which implies a binomial process. A test can then be set up based on the binomial distribution to determine the statistical significance of the empirically correct classification percentage. Since the percentage of correct classifications cannot deviate significantly below the expected value, a one tailed hypothesis test is most appropriate. The hypotheses to be tested are:

H_0: $P = P^e$

H_1: $P > P^e$

where:

P is the proportion of successes (correct classifications) resulting from the experiment.

P^e is the expected proportion of successes.

The sample size for all tests is large enough that the binomial distribution can be approximated by a normal distribution. The resulting decision rule is then:

Reject H_0 if and only if $P \geq P^e + Z_\alpha \sigma$

where:

P^e is the expected proportion of successes.

α is the chosen significance level.

Z is the standardized normal deviate for the desired significance level.

σ is the standard deviation of the binomial distribution.

The decision rule will be applied to each discriminate analysis to determine statistical significance. Rejection of the null hypothesis will effectively insure that the classification percentage is greater than a chance result. Non-rejection of the null hypothesis, however, will not insure that no differences exist. There is no knowledge of the power of the discriminate analysis, therefore, a lack of discrimination may have resulted from a lack of sensitivity to small differences. A complete examination of the classification table will be used to augment the analysis.

Identifying the principle causes of differentiation is an additional concern. Although differentiation may be found in a four factor model, it will be of interest to isolate the dominate contributor. Here again the normal procedures for isolating the dominate factor are not applicable. One approach would be to build a model for each combination of the factors. This would entail the analysis of twenty-three different models for each of the two holding periods being considered.[9] The confusion generated from this far exceeds the benefits that could be derived. Therefore, the search for the primary cause of differentiation will begin with an evaluation of the single factor models. Only if this does not produce satisfactory results will the two and three factor models be considered. Once the primary characteristics are isolated, the means of each marketability group will be examined in an effort to determine the direction of these differences.

Portfolio Analysis

Another area of interest involves the effects on systematic risk of differences associated with marketability. The most common approach to isolating systematic risk is with the use of a market model. This is not appropriate in this instance since it assumes a homogeneous market. Instead, a technique used by Evans and Archer is employed.[10] To examine the effect of naive diversification on portfolio variance Evans and Archer looked at the standard deviation of portfolios containing from one to forty securities. As the portfolios become larger more and more of the unsystematic variation was eliminated. The technique does not rely on normality or homogeneity across the entire market. This

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[9] Since there are five characteristics, of which two are mutually exclusive, the possible combinations allow for five one factor, nine two factor, seven three factor, and two four factor models.

[10] J. H. Evans and S. H. Archer, "Diversification and the Reduction of Dispersion: An Empirical Analysis," Journal of Finance, 23:761-767, December 1968.

technique is applied to the sample in the sample in the present study to evaluate each of the five characteristics described earlier. The study of naively diversified portfolios will serve two purposes. Since unsystematic factors dominate the ex post results for a firm's securities, they may mask small differences caused by marketability. Forming portfolios to remove most of the unsystematic factors should allow those small differences to become discernable. This, obviously, will not be effective if the effects of marketability are themselves unsystematic. Unsystematic effects of marketability found in the individual securities are less important since they could be easily eliminated in any portfolio. If the effects persist in larger portfolios, the relative importance of such findings are considerably greater.

The second purpose of studying the portfolios centers on the effectiveness of diversification under different marketability conditions. Evans and Archer found that naive diversification eliminated essentially all unsystematic risk with ten-security portfolios. It is possible that marketability affects the speed of risk reduction. Such differences will be observable, if present, in the graphical analysis.

In carrying out the portfolio construction each of the four marketability groups will be considered separately. Within each group, twenty random portfolios will be generated for each size from one to thirty. A discriminate analysis will then be performed for each portfolio size to determine if differences become stronger or weaker. The mean level of each characteristic will then be plotted for each group at each portfolio size. This will allow an easy determination of the magnitude and direction of any differences that are found.

Summary

The data for the present study consists of two matched samples, one consisting of daily returns, the other monthly returns. The two samples each contain returns for four hundred firms broken down into four groups of one hundred firms, with each group representing a different level of marketability. The primary statistical technique will be a modified discriminate analysis.

The analysis to be performed will attempt to provide answers to several questions. First, are there differences in the distributional characteristics of security returns that can be attributed to differences in marketability? If there are, the characteristics affected and the direction in which the effect occurs will be determined. Second, is the length of the holding period important in the existance and type of effects? Third, is the nature of any differences systematic? In this third area two questions will be examined. Are there differences that exist but that are masked by non-systematic factors? Do differences persist in a systematic form or are they non-systematic in origin and thus eliminated in a portfolio?

The tests are multifaceted to isolate any effects marketability has on return characteristics. The results of the tests and the answers to these questions are discussed in the next chapter.

Chapter IV

EMPIRICAL ANALYSIS

The general belief found in most investment texts[1] is that investors require a premium for lack of marketability. The lack of marketability becomes an element of risk. This is intuitively appealing and does have some empirical support.[2] As outlined previously there is also some conflicting evidence on the effects of marketability. These previous studies of return distributions indicate returns on securities with low marketability do not exhibit different means than those with high marketability, but do exhibit higher variability, positive skewness, and leptokurtosis.[3] Limitations of that earlier research, however, reduce the degree of confidence that one can place in generalizing from their results.

In this chapter, the effects of marketability on return distributions are addressed through the use of discriminant analysis to examine

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[1] For examples, Seha M. Tinic and Richard West, Investing in Securities: An Efficient Markets Approach, Addison-Wesley Publishing Co., Reading, Mass., 1979, p. 14 and Jerome B. Cohen, Edward D. Zinbarg, and Arthur Zeikel, Investment Analysis and Portfolio Management, revised edition, Irwin Press, Homewood, Illinois, 1973, p. 755.

[2] Lawrence Fisher, "Determinants of Risk Premiums on Corporate Bonds," Journal of Political Economy, 67:217-237, June 1959.

[3] Kalman J. Cohen, Walter L. Ness, Jr., Hitoshi Okuda, Robert A. Schwartz, and David K. Whitcomb, "The Determinants of Common Stock Returns Voliatility: An International Comparison," Journal of Finance, 31:733-740, May 1976.

the first four moments of the distributions plus a fifth measure related to the shape of the distributions. Both daily and monthly security returns are examined to determine the significance of holding period length on those effects. Because investors may not be interested in the distribution of returns on individual securities, the analyses are replicated over portfolios of various sizes.

The results of each analysis will be presented in a paired format, the results for the daily data followed by those for the monthly data.

The previous chapter described the sample selection procedure. Except for the biases described there that affect the entire sample, the four hundred securities should be representative of all securities which trade in the organized exchanges. For this same reason, the four subgroups should be homogeneous with respect to each other, except for marketability. Implicit in the analyses which follow is that assumption of homogeneity except for marketability.

Because marketability, or the lack of it, has been viewed as a risk component to the investor, preliminary tests of the market's response to the differences in marketability in the sample were conducted. With later tests to be concerned with the moments of the return distributions, these tests centered on an overall measure of the valuation process, the price-earnings (P-E) ratio. The desired ratio would use investors' expectations, but these are not measurable. Average ex post P-E ratios were computed by dividing the average month end price by the average annual earnings over the five year period. The first test was an analysis of variance. Because the sample groupings were based solely on marketability, homogeneity between groups would not be expected if investors perceive marketability as affecting risk and thus P-E ratios.

The analysis of variance resulted in an F ratio of 1.01 which was insignificant at the .05 level. The model that resulted explained less than one percent of the total variation in the P-E ratios. The data was also subjected to a discriminant analysis of the type described in the previous chapter. The results are shown in Table 2. The sample was correctly classified 26.5 percent of the time. The minimum classification ability for statistical significance is 28.6 percent at a significance level of .05.[4] Therefore the null hypothesis of homogeneity is supported. It should be noted that the third group captured the vast bulk of the classifications. This appeared to be the result of major differences in the variance-covariance matrices for the groups. It does not present any significant problems in this situation since we are only interested in classification ability.

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[4]The rational for using a normal approximation to the binominal distribution is presented in Chapter III. For all of the discriminant tests involving the four hundred individual securities, the following computations are appropriate.

Given:
$n = 400$ (sample size)
$p = .25$ (probability of correct classification by chance)

The mean and standard deviation of this binomial distribution are 100 and 8.6603 respectively. When converted to percentages they become 25 and 2.165. These values can then be converted to critical values as follows:

Significance Level	Critical Classification Percentage
.05	$25 + (2.165 \times 1.645) = 28.6$
.01	$25 + (2.165 \times 2.326) = 30.0$

Table 2

Classification of Sample Based on P-E Ratios (N = 400)

	Classified into Group				
From Group	1	2	3	4	Total
(Low) 1	6	1	88	5	100
2	6	1	77	16	100
3	7	1	84	8	100
(High) 4	7	2	76	15	100
Total	26	5	325	44	400

Since the P-E ratio is valid as a measure of risk only when the earnings are positive, a more legitimate test would be to exclude those firms with negative P-E ratios. After the resultant removal of from six to eight firms from each group, the new analysis of variance found an F ratio of 2.56 which is significant at the .05 level. However, that model accounted for just 2.8 percent of the total variability. The results of the revised discriminant analysis are in Table 3. Again, the third group drew a large percentage of the classifications, although not to the previous extent. The classification process was only 26.34 percent accurate, approximately the same as the first test and again insignificant.

Table 3

Classification of Sample Based on
P-E Ratios (N = 372)

		Classified into Group			
From Group	1	2	3	4	Total
(Low) 1	9	4	72	8	93
2	13	11	57	12	93
3	12	8	64	8	92
(High) 4	19	16	45	14	94
Total	53	39	238	42	372

Table 4

Mean P-E Ratios
(N = 372)

Group	Mean
(Low) 1	7.99
2	9.15
3	8.17
(High) 4	10.27

P-E ratios for each group are shown in Table 4. Visual examination of these does not support the null hypothesis of no differences between groups. Nor, however, does it indicate a uniform, direct relationship with marketability.

The tests indicate that the market does respond to differences in marketability. That response is weak, however, and appears to take the form of a reward for high marketability rather than the penalty for low marketability suggested in the investment texts.

With evidence that the market does perceive differences in marketability, the question becomes what effects marketability has on the distributions of returns. As stated, the particular concern is with the first four moments of those distributions and with the studentized range.

Four Factor Models

The first step in evaluating the effects of marketability on return distributions is to determine if any effects exist. To this end, two four factor multivariate discriminant tests were performed on the daily data. The first test was based on the first four moments, with resulting classifications as shown in Table 5. There were 162 correctly classified firms, which represents 40.5 percent of the sample. Groups 1, 2, and 3 tended to be classified as group 2 while the high marketability group, 4, was identified as a separate group.

Table 5

Classification of the Daily Sample Based on the First Four Moments (N = 400)

	Classified into Group				
From Group	1	2	3	4	Total
(Low) 1	15	50	16	19	100
2	4	62	11	22	100
3	10	49	24	17	100
(High) 4	6	21	12	61	100
Total	35	182	63	120	400

The test was repeated substituting the studentized range for the fourth moment. The results were very similar with 156 (39 percent) of the firms correctly classified. The classification table for this second test is shown in Table 6.

Table 6

Classification of the Daily Sample Based on the First Three Moments and the Studentized Range (N = 400)

	Classified into Group				
From Group	1	2	3	4	Total
(Low) 1	24	49	11	16	100
2	10	65	10	15	100
3	19	46	21	14	100
(High) 4	11	31	12	46	100
Total	64	191	54	91	400

Neither of these tests resulted in what would be considered a good classification ability. However, both are statistically significant at the .01 level. It is impossible to attribute the difference that has been found to any particular factor at this point.

These two tests were replicated on the monthly returns, with similar results. On the basis of four moments, 164 firms were correctly classified, 41 percent of the sample. When the studentized range was substituted for the fourth moment, the correct classification was 43.25 percent (173 firms). Both of these results are statistically significant and comparable to what was obtained with the daily returns. Again groups 1, 2, and 3 were found to be similar while group 4 was quite separated from them.

The continued ability to discriminate based on monthly holding periods is itself a significant result. Marketability is naturally a function of the length of the holding period. Differences in return distributions due to differences in marketability are, however, apparently not dependent on the length of the holding period.

One Factor Models

To isolate the most significant element responsible for the differences, each of the five distributional characteristics was examined separately. The average values for each of those characteristics of the daily return distributions for each of the four groups are shown in Table 7. The average values using monthly returns are shown in Table 8.

Mean

A discriminant analysis performed on the means of the daily returns resulted in just 29.25 percent (117 firms) being correctly classified, a statistically significant difference at the .05 level, but not at the .01 level. An analysis of variance test produced an F ratio of 5.76 which is significant at the .01 level. When the tests were repeated with the monthly data, 126 firms (31.5 percent) were correctly classified, significant at the .01 level. The analysis of variance F ratio of 2.24 is not significant at the .05 level. The apparent conflict between the two tests on the monthly data can be explained by the fact that in the discriminant analysis over 50% of the observations were classified into group 1. Out of the 126 correct classifications, 62 were the result of observations from group 1 being classified into group 1. This tends to invalidate the significance of the classification ability. The lack of difference in the means between groups for both holding periods suggests three possible explanations. First, the market is inefficient, in a risk-return sense, with respect to the effect of marketability. Second, the market is efficient, but the risk-return tradeoff is a step or non-uniform function. Third, the risk differences are and can be eliminated by the use of diversification. These alternatives are examined in the section of this chapter on portfolio analysis.

Standard Deviation

A study of the standard deviations revealed somewhat improved classification ability. The sample based on daily returns produced a 35.5 percent classification ability which is significant at a .01 level. The F ratio for the analysis of variance test was 6.54, significant at the .01 level. Thus the standard deviation does contribute to the overall differences that were found. It would appear that trading activity and variance are directly related except when trading is very small. In that case variability increases.

Based on monthly returns the classification ability of the standard deviation increased to 40.5 percent (162 firms). This is approximately the same percentage as was achieved using all four characteristics. As with the means, groups 1, 2, and 3 were very similar, while group 4 was distinctly different. An analysis of variance test also showed extremely strong significance with an F ratio of 17.21. The pattern of the relationship between marketability and standard deviation changed

Table 7

Average Values for Distribution Statistics
Daily Returns

Marketability Group	Mean	Standard Deviation	Skewness	Kurtosis	Studentized Range
(Low) 1	.05%	2.25%	.244	4.73	8.24
2	.07	2.04	.224	2.77	7.67
3	.05	2.25	.198	3.04	7.70
(High) 4	.02	2.72	.237	2.80	7.74

Table 8

Average Values for Distribution Statistics
Monthly Returns

Marketability Group	Mean	Standard Deviation	Skewness	Kurtosis	Studentized Range
(Low) 1	1.18%	9.84%	.533	2.47	5.71
2	1.06	9.98	.418	1.73	5.46
3	1.36	10.92	.375	1.27	5.41
(High) 4	1.36	13.16	.319	1.62	5.54

slightly from what was observed with a daily holding period, as shown in Table 8. The direct relationship between trading activity and standard deviation exists across all four groups.

Skewness

The discriminant analysis was next performed on the third moment, skewness. Daily data resulted in 104 (26 percent) of the firms being correctly classified, very close to the 100 expected by random chance and thus not significant. Most of the observations were classified into group 4 which verifies the lack of differentiation. The test was then applied to the monthly data, with only slightly better results. In this case 126 firms, representing 31.5 percent of the sample, were correctly classified. This value is not indicative of a strong non-random separating ability but is significant of the .01 level. Here again, most of the observations were classified into one group. In this case group 3. The increased classification ability is a result of groups 1 and 4 showing a moderate number of correct classifications. The F ratio for the daily and monthly holding periods were .13 (non-significant) and 2.82 (significant at .05) respectively. The increase in classification ability as the holding period was lengthened has two possible explanations. It may indicate the existence of a relationship between the length of the holding period and the skewness characteristics of the return distributions. An alternative explanation is that measurement error is reduced. However, these possiblities will not be explored further in this study.

Kurtosis and Studentized Range

An examination of the impact of kurtosis provided little additional explanatory information. Both the fourth moment and the studentized range measure the extent of kurtosis. Using the fourth moment of the daily return distributions resulted in a 25 percent classification rate. Group 4 captured 319 out of the 400 observations, indicating no discriminanting ability. The studentized range, which centers on the tails of the distributions, produced a correct classification for 121 firms, 30.25 percent of the sample and significant at the .01 level.

Although group 4 again captured over fifty percent of the observations, groups 1 and 2 showed an improved classification ability. Group 3 appeared to be effectively absorbed into group 4. When the analysis of variance test was run on the two measures, F ratios of 2.64 for the fourth moment and 3.36 for the studentized range were obtained. Both ratios are significant at the .05 level. It would appear that any difference in kurtosis that exists is more evident in the tails than around the mean. When the monthly holding periods were examined, the classification abilities of the two kurtosis measures were closer, but still not high. The fourth moment resulted in a 32.25% correct classification rate significant at the .01 level. The studentized range classified 29.25 percent of the sample correctly. Group 3 had the largest number of classifications for both measures. When the fourth moment was tested no other group showed a strong number of classifications. The F ratios were 6.49 for the fourth moment and 3.30 for the studentized range. These are significant at the .01 and .05 levels respectively. The results of all four of these tests of kurtosis are not impressive. An examination of Tables 7 and 8 indicated that there is a weak tendency for the degree of leptokurtosis to decline as marketability increases. This tendency is particularly evident when moving from group 1 to group 2. Several of the studies reviewed in Chapter II also found this relationship.

In summary, the return distributions do appear to be a function of marketability primarily through the standard deviations of the distributions. The other distributional measures that were examined revealed a much weaker relationship with marketability.

Portfolio Analysis

Current investment theory indicates that the only those attributes of a stock which cannot be eliminated through diversification are relevant. To examine the systematic nature of the marketability effects, random portfolios were created from each marketability group. The portfolios ranged in size from one to thirty securities. A total of twenty portfolios were generated for each marketability group and each size group.

Four Factor Models

Initially, two discriminant models were considered for the daily returns. One was based on the four moments. A summary of those results is shown in Table 9. The results from the small portfolios are higher than those reported in the previous section for the same model, the result of sampling error. As the number of securities in the portfolio increases, the classification ability improved markedly. Apparently, as the portfolio size increased, naive diversification removed a great deal of the unsystematic differences, leaving differences that could be explained as being related to marketability. The previous analysis of P-E ratios suggests that the only difference between groups is marketability. The classification ability with larger portfolios leaves no doubt that identifiable differences exist between the groups.

The second model was constructed by replacing the fourth moment with the studentized range. As shown in Table 10, the results again showed a definite increase in classification ability as the portfolio size increases.

Table 9

Discriminant Analysis for Daily Portfolio Returns
Based on Mean, Standard Deviation, Skewness, and Kurtosis

Portfolio Size	Percent Correctly Classified	Portfolio Size	Percent Correctly Classified
1	51.25	16	92.50
2	47.50	17	83.75
3	63.75	18	87.50
4	60.00	19	87.50
5	65.00	20	88.75
6	70.00	21	85.00
7	67.50	22	93.75
8	80.00	23	92.50
9	76.25	24	90.00
10	77.50	25	91.25
11	75.00	26	90.00
12	80.00	27	95.00
13	78.75	28	93.75
14	87.50	29	95.00
15	80.00	30	97.50

Table 10

Discriminant Analysis for Daily Portfolio Returns Based
on Mean, Standard Deviation, Skewness, and Studentized Range

Portfolio Size	Percent Correctly Classified	Portfolio Size	Percent Correctly Classified
1	53.75	16	88.75
2	53.75	17	83.75
3	60.00	18	88.75
4	61.25	19	88.75
5	56.25	20	90.00
6	61.25	21	83.75
7	77.50	22	92.50
8	78.75	23	91.25
9	77.50	24	88.75
10	78.75	25	88.75
11	75.00	26	88.75
12	77.50	27	95.00
13	80.00	28	92.50
14	82.75	29	96.25
15	80.00	30	98.50

Table 11

Discriminant Analysis for Monthly Portfolio Returns Based on Mean, Standard Deviation, Skewness, and Kurtosis

Portfolio Size	Percent Correctly Classified	Portfolio Size	Percent Correctly Classified
1	50.00	16	83.75
2	55.00	17	87.50
3	57.50	18	87.50
4	65.00	19	81.25
5	61.25	20	95.00
6	67.50	21	90.00
7	65.00	22	96.25
8	71.00	23	92.50
9	73.75	24	90.00
10	80.00	25	97.50
11	77.50	26	96.25
12	81.25	27	98.75
13	80.00	28	92.50
14	87.50	29	98.75
15	86.25	30	91.25

Table 12

Discriminant Analysis for Monthly Portfolio Returns Based on Mean, Standard Deviation, Skewness, and Studentized Range

Portfolio Size	Percent Correctly Classified	Portfolio Size	Percent Correctly Classified
1	43.75	16	88.75
2	55.00	17	87.50
3	56.25	18	85.00
4	65.00	19	85.00
5	60.00	20	96.25
6	68.75	21	90.00
7	61.25	22	96.25
8	53.75	23	91.25
9	70.00	24	90.00
10	82.50	25	97.50
11	82.50	26	97.50
12	81.25	27	97.50
13	78.75	28	92.50
14	86.25	29	100.00
15	87.50	30	95.00

These two models were next applied to portfolios constructed from the monthly holding period return sample. As can be seen in Tables 11 and 12, these analyses produced results that are very similar to those just discussed. The maximum classification ability is slightly higher than with the daily returns, but the increase does not warrant any conclusion with respect to the relevance of the holding period on the effects of marketability.

Although these results are very promising, caution should be exercised with respect to overstating their significance at this point. It is impossible to determine what level of marketability is "best" or even if there is such a situation. In order to isolate the cause of the discriminating ability, each of the distributional characteristics must be examined separately.

One Factor Models

Mean

When the means alone were examined, the results were mixed. Table 13 summarizes the classification ability when the analysis was applied to the daily returns. The accuracy of the model increased as the portfolio size increased from one to twenty-four. Beyond this, the classification ability seems to decline to some extent.

The mean of the means for each group at each portfolio size were examined. This data is plotted in Figure 1. The means fell into two ranges. These groupings do not seem logical, however. The higher range contains the lowest marketability group and the next to highest group, while the lower range contains the highest and next to lowest marketability groups.

Table 13

Discriminant Analysis for Daily Portfolio Returns
Based on Means

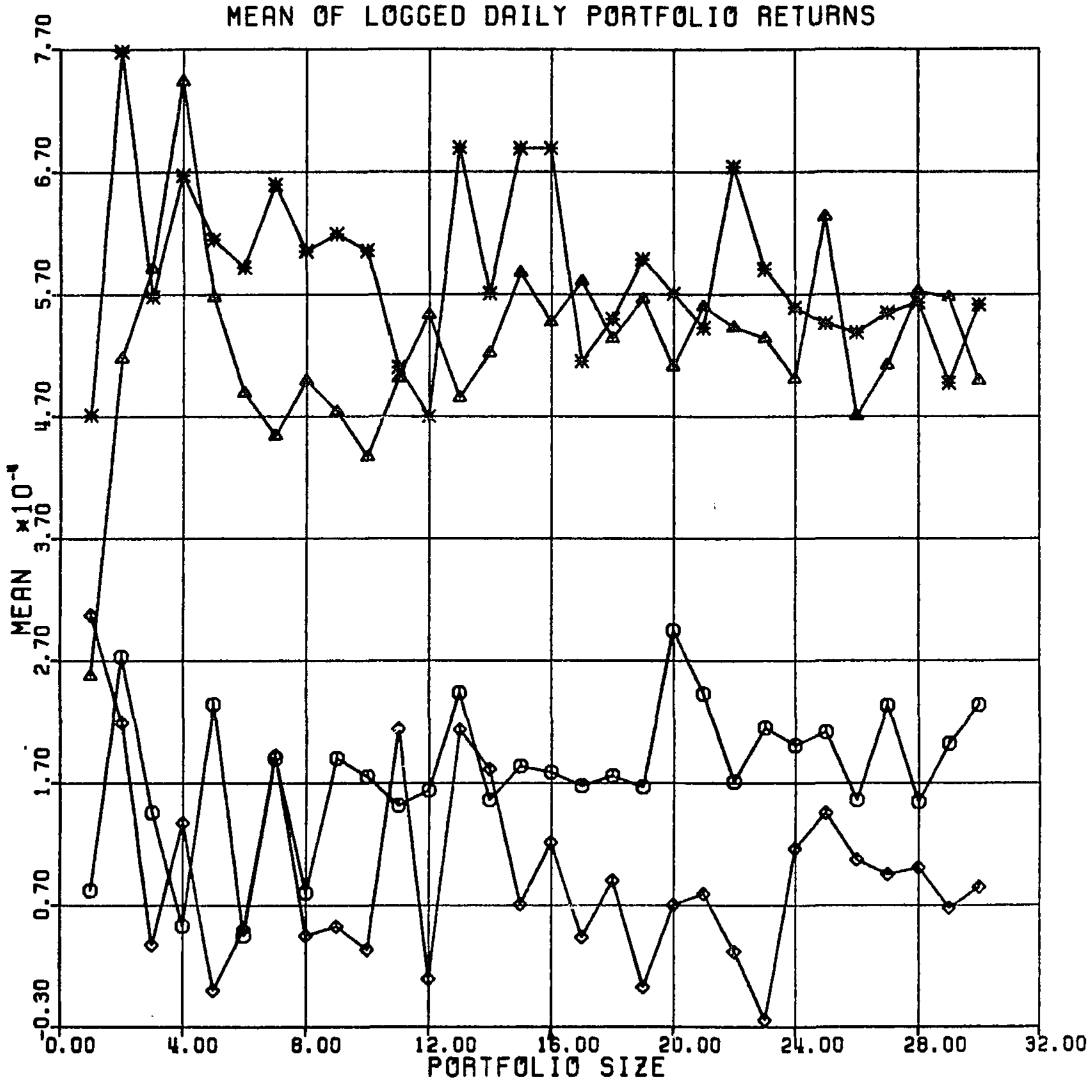
Portfolio Size	Percent Correctly Classified	Portfolio Size	Percent Correctly Classified
1	30.00	16	52.50
2	27.50	17	41.25
3	33.75	18	40.00
4	38.75	19	50.00
5	40.00	20	48.75
6	36.25	21	53.75
7	35.00	22	55.00
8	41.25	23	52.50
9	48.75	24	53.75
10	38.75	25	50.00
11	37.50	26	47.50
12	52.50	27	58.75
13	45.00	28	43.75
14	38.75	29	48.75
15	50.00	30	46.25

Table 14

Discriminant Analysis for Monthly Portfolio Returns
Based on Means

Portfolio Size	Percent Correctly Classified	Portfolio Size	Percent Correctly Classified
1	31.25	16	30.00
2	35.00	17	40.00
3	40.00	18	38.75
4	32.50	19	45.00
5	31.25	20	47.50
6	38.75	21	43.75
7	30.00	22	45.00
8	28.75	23	43.75
9	41.25	24	42.50
10	32.50	25	42.50
11	33.75	26	37.50
12	43.75	27	38.75
13	37.50	28	55.00
14	37.50	29	40.00
15	42.50	30	42.50

Figure 1



Symbol	Marketability Group
✱——✱	1 (Low)
◇——◇	2
△——△	3
○——○	4 (High)

The unusual pairing was not supported by the results of the analysis of monthly returns. Table 14 shows that the discriminant function for the monthly returns reached a classification ability in the mid forties, which is significant at the .01 level.[5] The classification ability is somewhat lower than what was achieved with the daily return sample. The plotting of the mean points, Figure 2, revealed a more plausable pattern than was exibited by the daily return sample. Again there was a tendency for the points to separate into two groups. The extent of this separation is much less and the membership is more logical. The two upper marketability groups had the highest returns, while the two lower groups had considerably lower returns. This would not be in line with a traditional risk-return relationship, where marketability reduces risk to the investor.

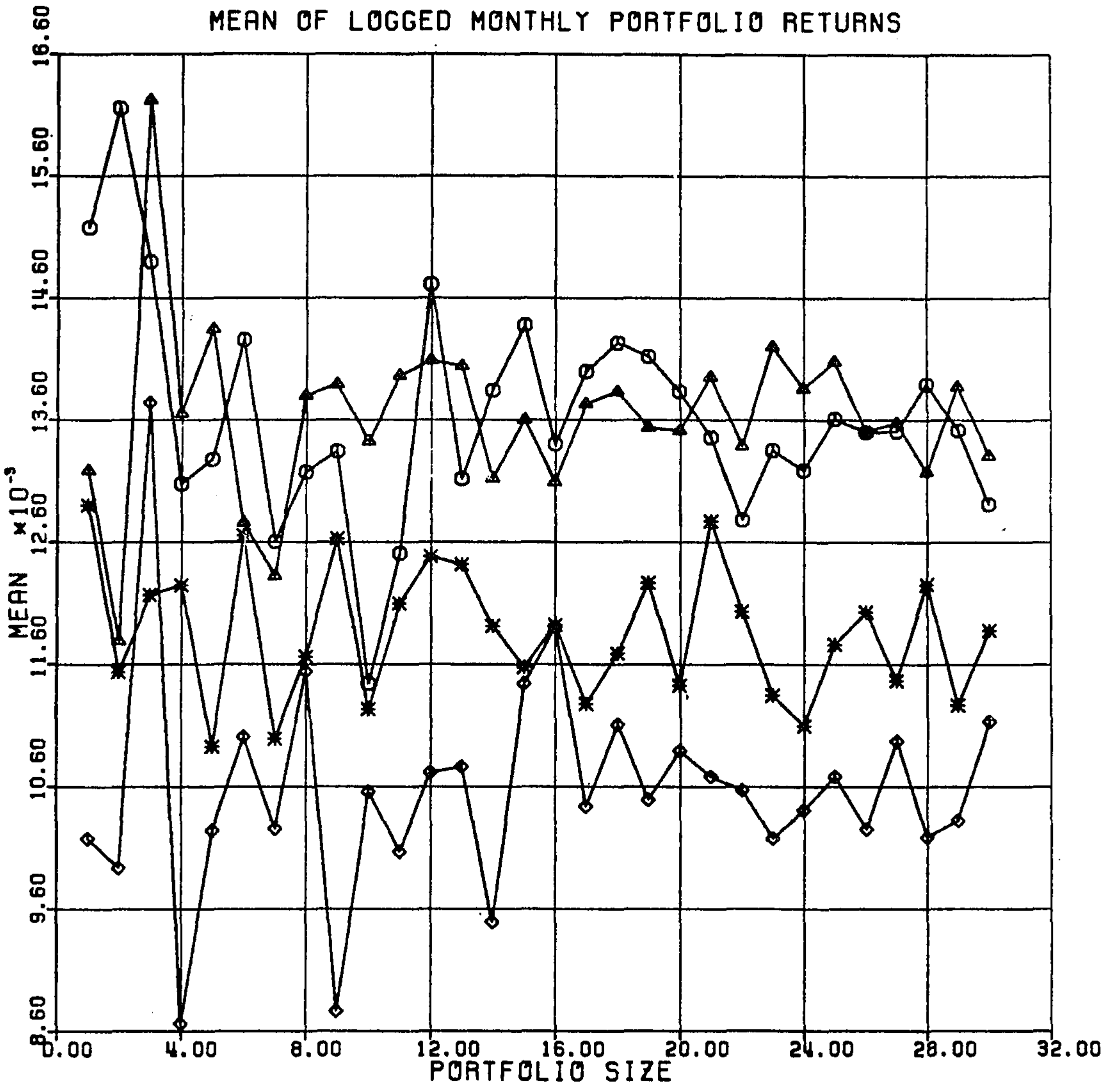
Standard Deviation

A study of the standard deviations reveals a risk pattern in line with the results of the study of the portfolio means. Tables 15 and 16 show that the discriminating ability of a model based on the standard deviation improves greatly as the portfolio size increases. As before, the monthly holding period results are superior to those based on a daily holding period.

The graph of the daily mean standard deviation (Figure 3) shows a more consistent pattern that the graph of means. The patterns are essentially in conformance with previous research on the effects of diversification on portfolio standard deviations, in that the standard

[5]For all of the portfolio studies (N = 80) statistical significance is achieved at 33 percent correct classification for a .05 level of significance and at 36 for a .01 level.

Figure 2



Symbol	Marketability Group
✱——✱	1 (Low)
◇——◇	2
▲——△	3
○——○	4 (High)

Table 15

Discriminant Analysis for Daily Portfolio Returns
Based on Standard Deviation

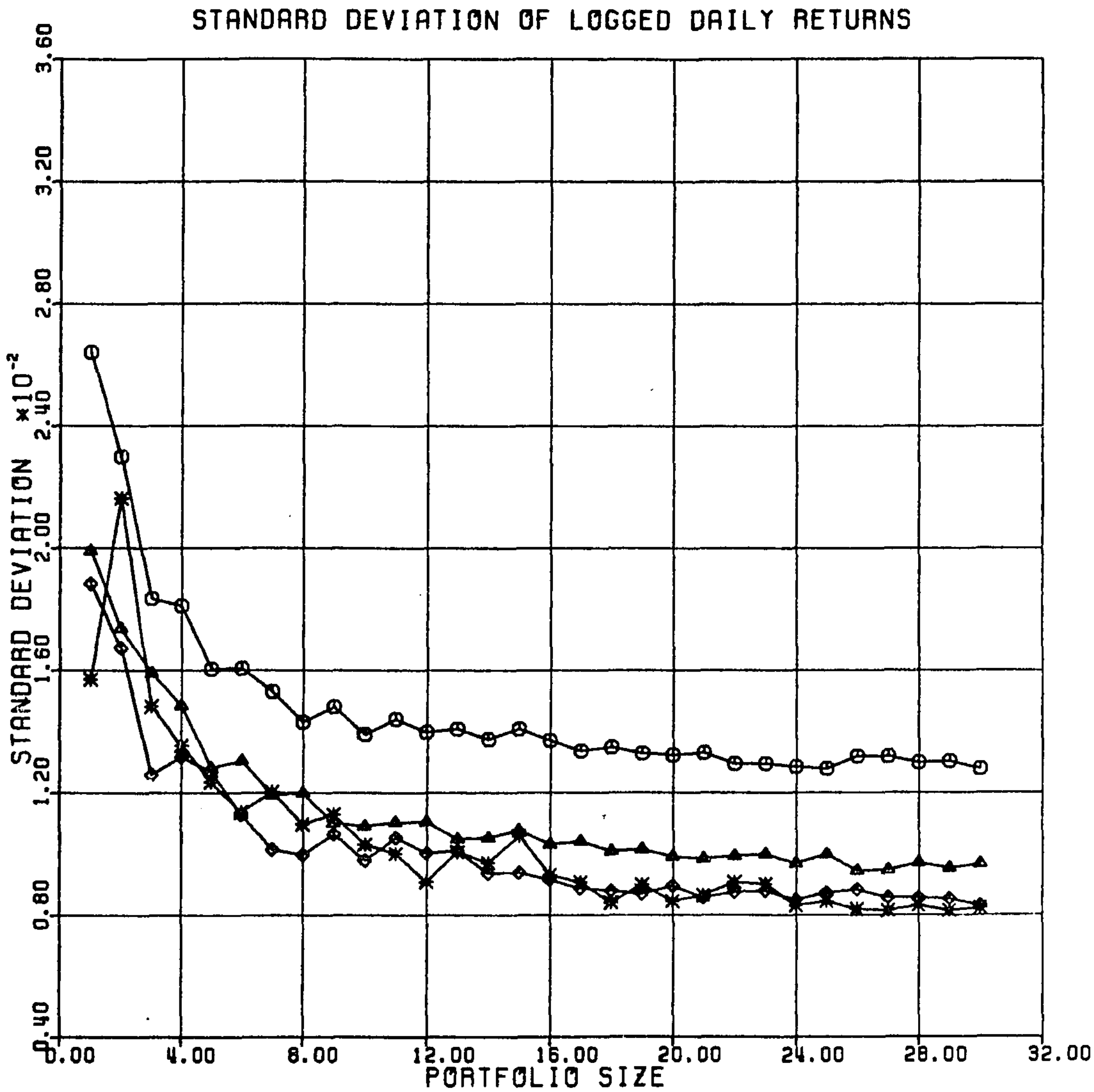
Portfolio Size	Percent Correctly Classified	Portfolio Size	Percent Correctly Classified
1	40.00	16	65.00
2	42.50	17	65.00
3	41.25	18	75.00
4	41.25	19	65.00
5	42.50	20	65.00
6	40.00	21	62.50
7	56.25	22	63.75
8	48.75	23	68.75
9	53.75	24	67.50
10	61.25	25	67.50
11	50.00	26	73.75
12	57.50	27	68.75
13	53.75	28	67.50
14	58.75	29	68.75
15	60.00	30	68.75

Table 16

Discriminant Analysis for Monthly Portfolio Returns
Based on Standard Deviation

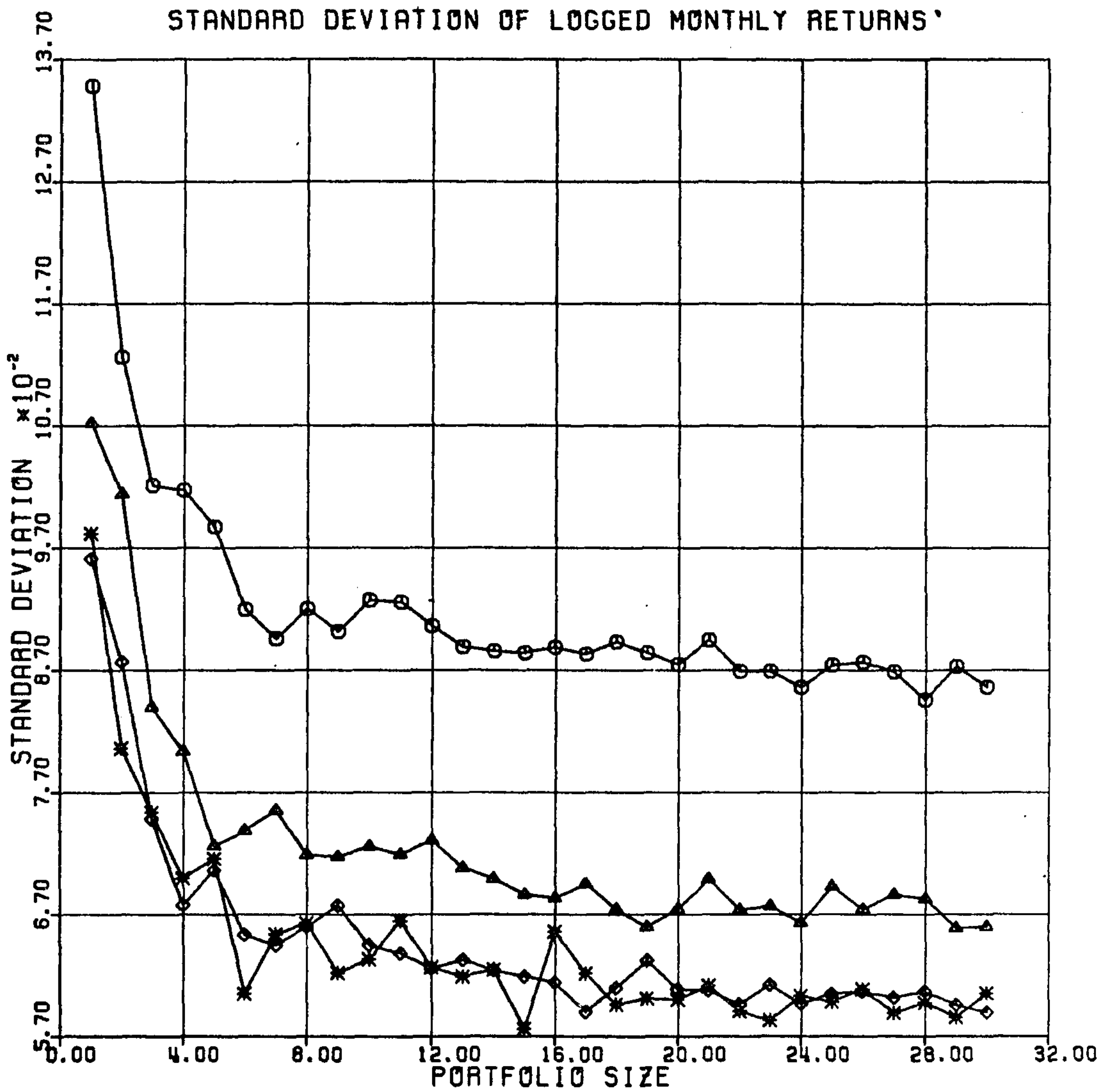
Portfolio Size	Percent Correctly Classified	Portfolio Size	Percent Correctly Classified
1	36.25	16	62.50
2	42.50	17	57.50
3	48.75	18	62.50
4	55.00	19	65.00
5	45.00	20	62.50
6	53.75	21	62.50
7	51.25	22	67.50
8	47.50	23	71.25
9	53.75	24	58.75
10	61.25	25	71.25
11	55.00	26	71.25
12	62.50	27	76.25
13	56.25	28	63.75
14	65.00	29	66.25
15	65.00	30	72.50

Figure 3



Symbol	Marketability Group
✱——✱	1 (Low)
◇——◇	2
▲——▲	3
⊖——⊖	4 (High)

Figure 4



Symbol	Marketability Group
✱——✱	1 (Low)
◇——◇	2
▲——▲	3
⊖——⊖	4 (High)

deviation decreases with larger portfolios.[6] The ordering of the groups is consistant with the marketability levels. The two lowest marketability groups show almost identical levels throughout most of the curve. At marketability levels above this, the standard deviation increases with added marketability. Figure 4, based on monthly holding periods, shows this relationship even more clearly. Again the two lowest marketability groups are practically indistinguishable; however, the two higher groups have more clearly separated themselves from the bottom groups. It is possible that with even longer holding periods the two lowest groups would separate and provided additional information about the effect of low marketability on portfolio standard deviation. On the basis of the results that have been presented, it would appear that there is a direct relationship between marketabilty and standard deviation.

Before examining any higher moments, some additional comments are in order. The results for the mean and standard deviation are internally consistent to the traditional risk-return relationship, at least for the monthly returns. Increases in standard deviation accompany increases in mean. There is some departure in the highest marketability group where an increase in mean return does not accompany an increase in standard deviation. One explanation is that the higher moments are relevant to the investors' risk-return tradeoffs. Another is that marketability may itself be considered a risk factor beyond the distributional characteristics. Intuitively marketability reduces

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[6]Lawrence Fisher and James Lorie, "Some Studies of Variability of Returns on Investments in Common Stocks," Journal of Business, 43:99-134, April, 1970.

Table 17

Discriminant Analysis for Daily Portfolio Returns
Based on Skewness

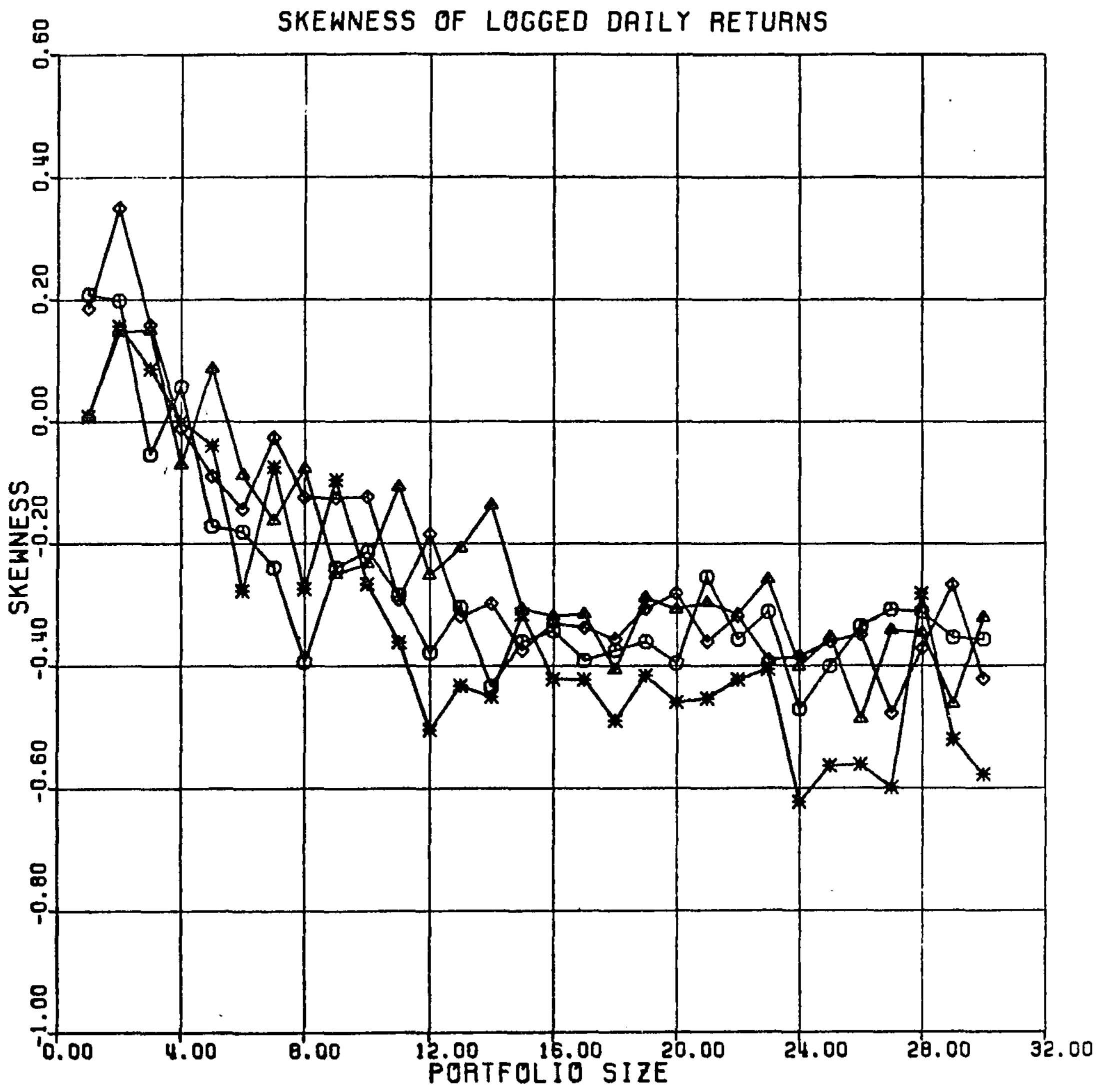
Portfolio Size	Percent Correctly Classified	Portfolio Size	Percent Correctly Classified
1	31.25	16	38.75
2	28.75	17	35.00
3	35.00	18	37.50
4	30.00	19	37.50
5	27.50	20	35.00
6	40.00	21	40.00
7	45.00	22	35.00
8	36.25	23	47.50
9	37.50	24	36.25
10	35.00	25	42.50
11	36.25	26	45.00
12	30.00	27	37.50
13	36.25	28	30.00
14	31.25	29	35.00
15	36.25	30	41.25

Table 18

Discriminant Analysis for Monthly Portfolio Returns
Based on Skewness

Portfolio Size	Percent Correctly Classified	Portfolio Size	Percent Correctly Classified
1	37.50	16	45.00
2	37.50	17	35.00
3	28.75	18	38.75
4	32.50	19	46.25
5	35.00	20	47.50
6	35.00	21	37.50
7	33.75	22	43.75
8	41.25	23	38.75
9	42.50	24	45.00
10	45.00	25	47.50
11	42.50	26	50.00
12	38.75	27	52.50
13	38.75	28	58.75
14	47.50	29	53.75
15	42.75	30	50.00

Figure 5



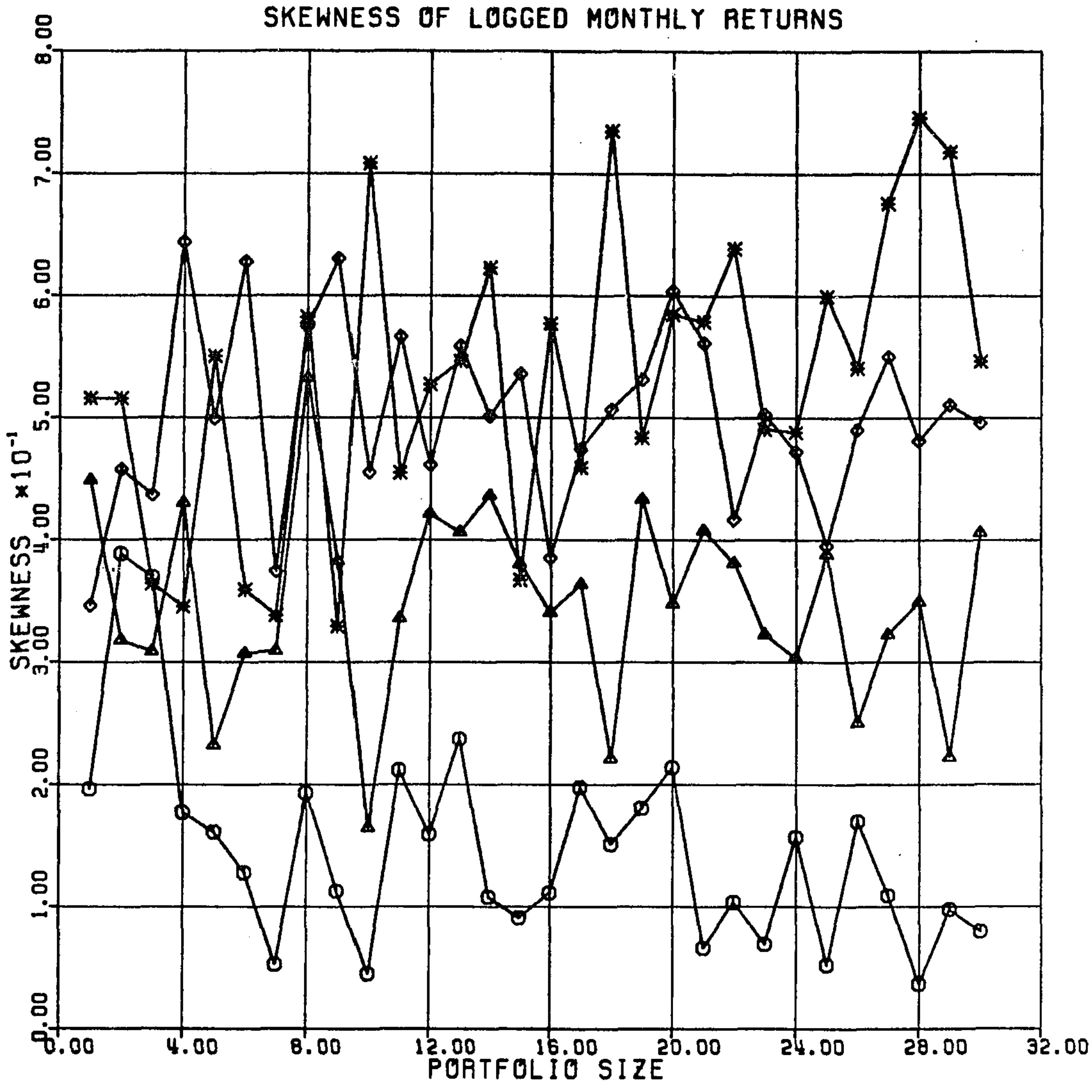
Symbol	Marketability Group
✱—✱	1 (Low)
◇—◇	2
▲—▲	3
⊖—⊖	4 (High)

investors' risk, consistent with the theories discussed in Chapter II in which thinness increased standard deviation of returns. The results for the standard deviations reported above, however, are contrary to that relationship. The standard deviation and mean are found to be directly related to marketability. Although ad hoc explanations for this results can be made, no theoretical justification is apparent.

Skewness

Continuing the examination of higher moments, the discriminant analysis on the skewness coefficient of daily returns showed statistically significant classification ability for portfolios containing six or more securities, but that ability is far lower than what was found for the standard deviation (Table 17). The graph in Figure 5 shows that the skewness coefficients overlap and maintain no consistent ordering. The rapid movement of this measure from positive to negative as portfolio size increases is interesting, because it is not repeated when the monthly holding period sample is studied (Figure 6). In all cases, for the daily holding period, the extent of skewness is small. There is no readily available explaination for the observed behavior of the skewness coefficient, thus it will be left to future research to determine if it can be replicated and explained. The monthly holding period, in general, provided more understandable results. The discriminant analysis (Table 18) revealed a greater classification ability, particularly with respect to the larger portfolios. The marketability groups are better separated than was the case with the daily sample, as shown in Figure 6. However, there does seem to be some problem with the stability of this measure of skewness, particularly for small portfolios ($N < 8$). This does not overshadow the relationship

Figure 6



Symbol	Marketability Group
✱—✱	1 (Low)
◇—◇	2
▲—△	3
○—○	4 (High)

Table 19

Discriminant Analysis for Daily Portfolio Returns
Based on Kurtosis

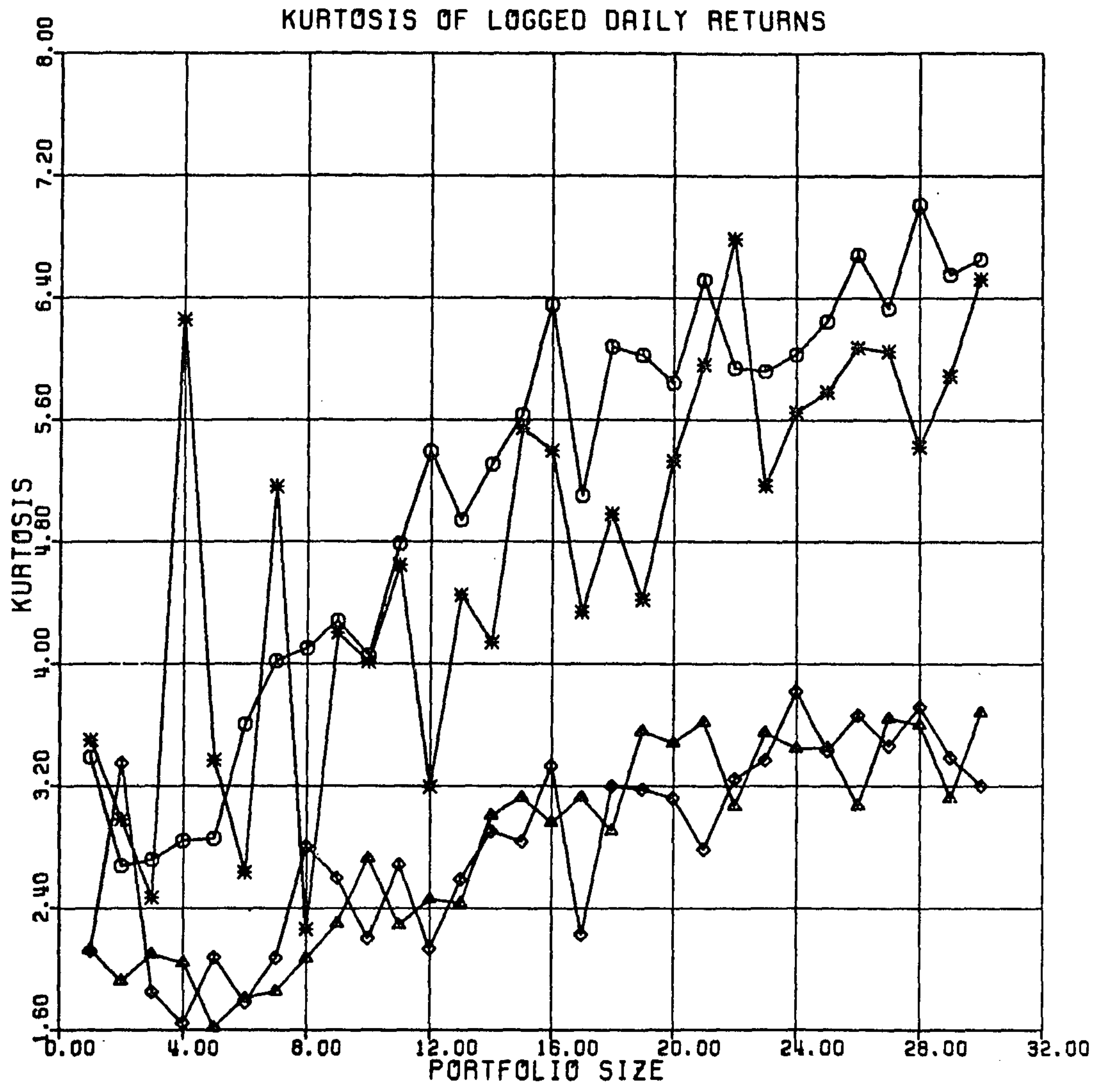
Portfolio Size	Percent Correctly Classified	Portfolio Size	Percent Correctly Classified
1	32.50	16	48.75
2	31.25	17	46.25
3	27.50	18	45.00
4	31.25	19	43.75
5	38.75	20	47.50
6	36.25	21	53.75
7	43.75	22	53.75
8	35.00	23	42.50
9	42.50	24	36.25
10	36.25	25	51.25
11	43.75	26	57.50
12	46.25	27	51.25
13	47.50	28	45.00
14	37.50	29	50.00
15	40.00	30	52.50

Table 20

Discriminant Analysis for Daily Portfolio Returns
Based on Studentized Range

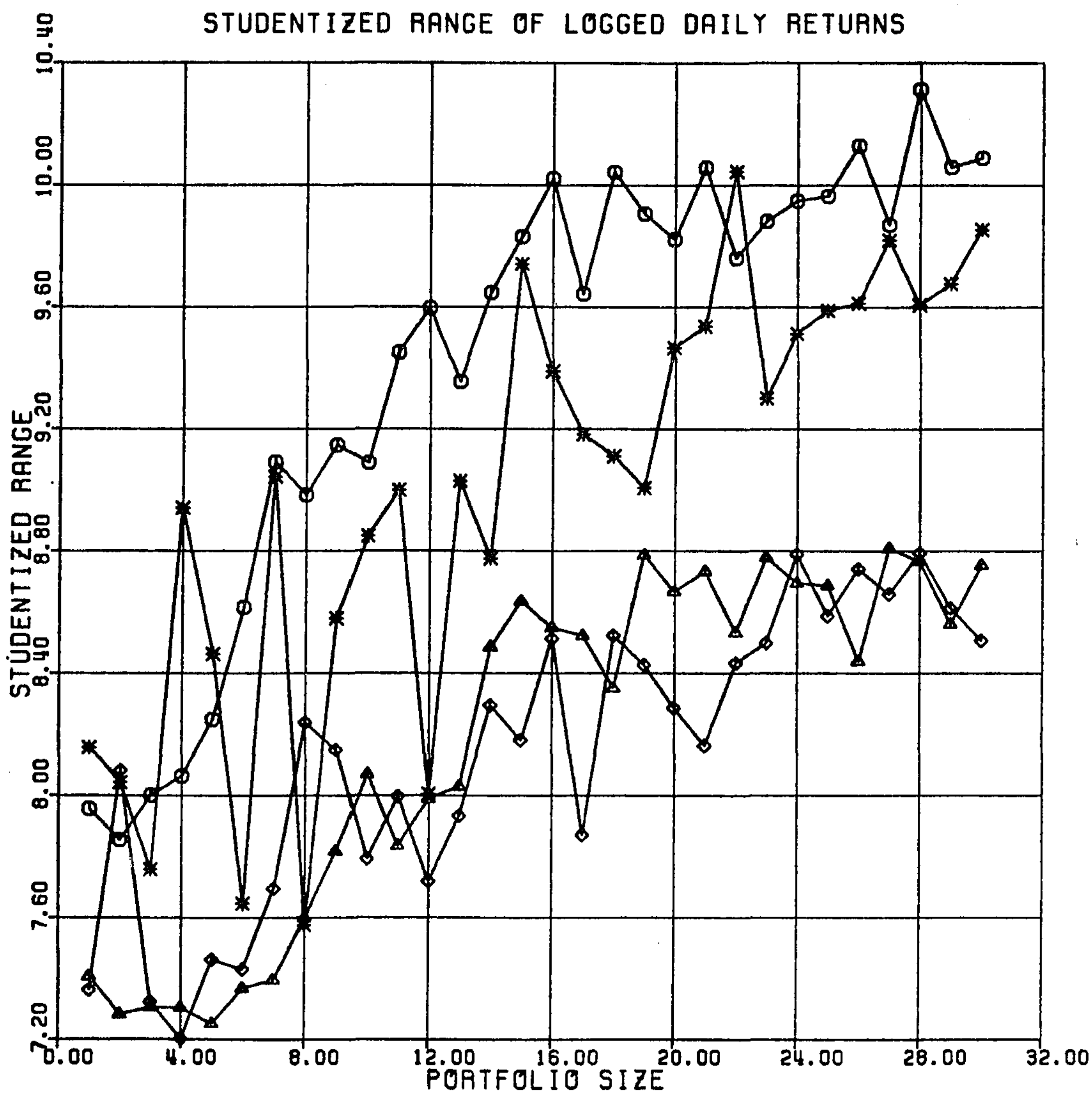
Portfolio Size	Percent Correctly Classified	Portfolio Size	Percent Correctly Classified
1	26.25	16	46.25
2	33.75	17	46.25
3	28.75	18	42.50
4	38.75	19	42.50
5	37.50	20	52.50
6	32.50	21	46.25
7	41.25	22	47.50
8	38.75	23	50.00
9	35.00	24	45.00
10	36.25	25	48.75
11	38.75	26	55.00
12	41.25	27	42.50
13	42.50	28	46.25
14	40.00	29	45.00
15	40.00	30	58.75

Figure 7



Symbol	Marketability Group
✱—✱	1 (Low)
◇—◇	2
▲—△	3
○—○	4 (High)

Figure 8



Symbol	Marketability Group
✱—✱	1 (Low)
◇—◇	2
▲—▲	3
○—○	4 (High)

that appears. None of the skewness levels are particularly large, but they definitely decrease as marketability increases, giving an indirect relationship between marketability and skewness.

An additional relationship is found when the effect of diversification is examined with respect to the level of marketability. The middle two groups seem to be uneffected by portfolio size. The high marketability group shows a definite decrease in the degree of skewness as portfolio size is increased. This relationship is in agreement with the results of a study done by Simkowitz and Beedles.[7] The lowest marketability group gives a slight indication of an opposite relationship, with skewness increasing for the largest portfolios.

Kurtosis and Studentized Range

Kurtosis of the portfolios was examined using both the kurtosis coefficient and the studentized range. A discriminant analysis was applied to both measures based on the sample of daily holding period returns. It can be seen from Tables 19 and 20 that very similar results were obtained from the two analyses. In both cases the classification ability moved from an insignificant level for small portfolios to over fifty percent for large portfolios which is significant at the .01 level. Figures 7 and 8 reveal that with both measures of kurtosis the group means tended to separate into two sets and increase as the portfolio size increased. The highest and lowest marketability groups had higher levels of kurtosis than the two middle groups. With respect to the lowest group the elevated kurtosis may be a result of many no-trade

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[7] Michael A. Simkowitz and William L. Beedles, "Diversification in a Three Moment World," Journal of Financial and Quantitative Analysis, 13:927-941, December 1978.

Table 21

Discriminant Analysis for Monthly Portfolio Returns
Based on Kurtosis

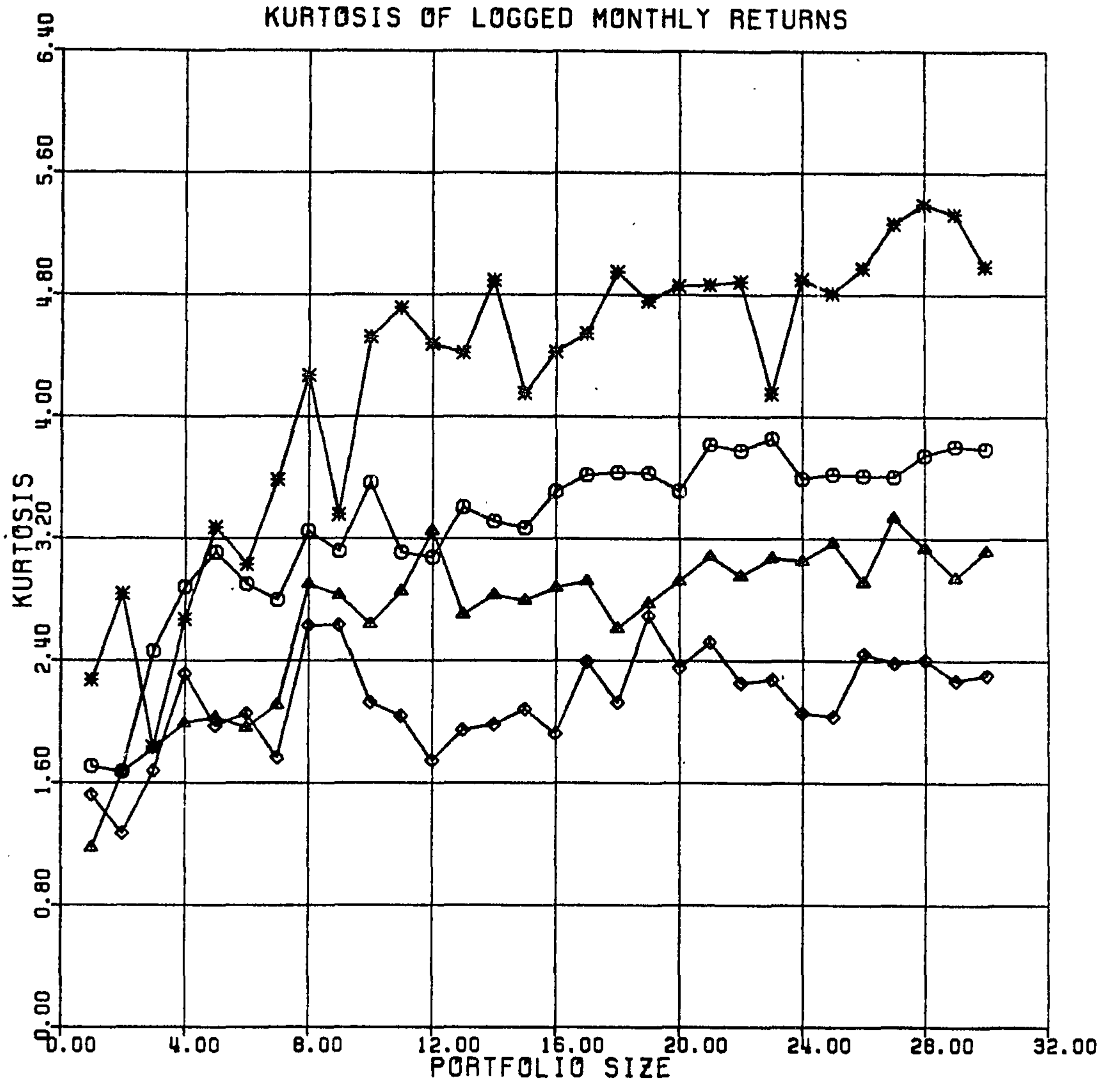
Portfolio Size	Percent Correctly Classified	Portfolio Size	Percent Correctly Classified
1	31.25	16	57.50
2	37.50	17	47.50
3	36.25	18	57.50
4	31.25	19	48.75
5	37.50	20	51.25
6	33.75	21	51.25
7	38.75	22	55.00
8	35.00	23	52.50
9	28.75	24	56.25
10	52.50	25	57.50
11	48.75	26	66.25
12	52.50	27	63.75
13	50.00	28	63.75
14	56.25	29	68.75
15	47.50	30	61.25

Table 22

Discriminant Analysis for Monthly Portfolio Returns
Based on Studentized Range

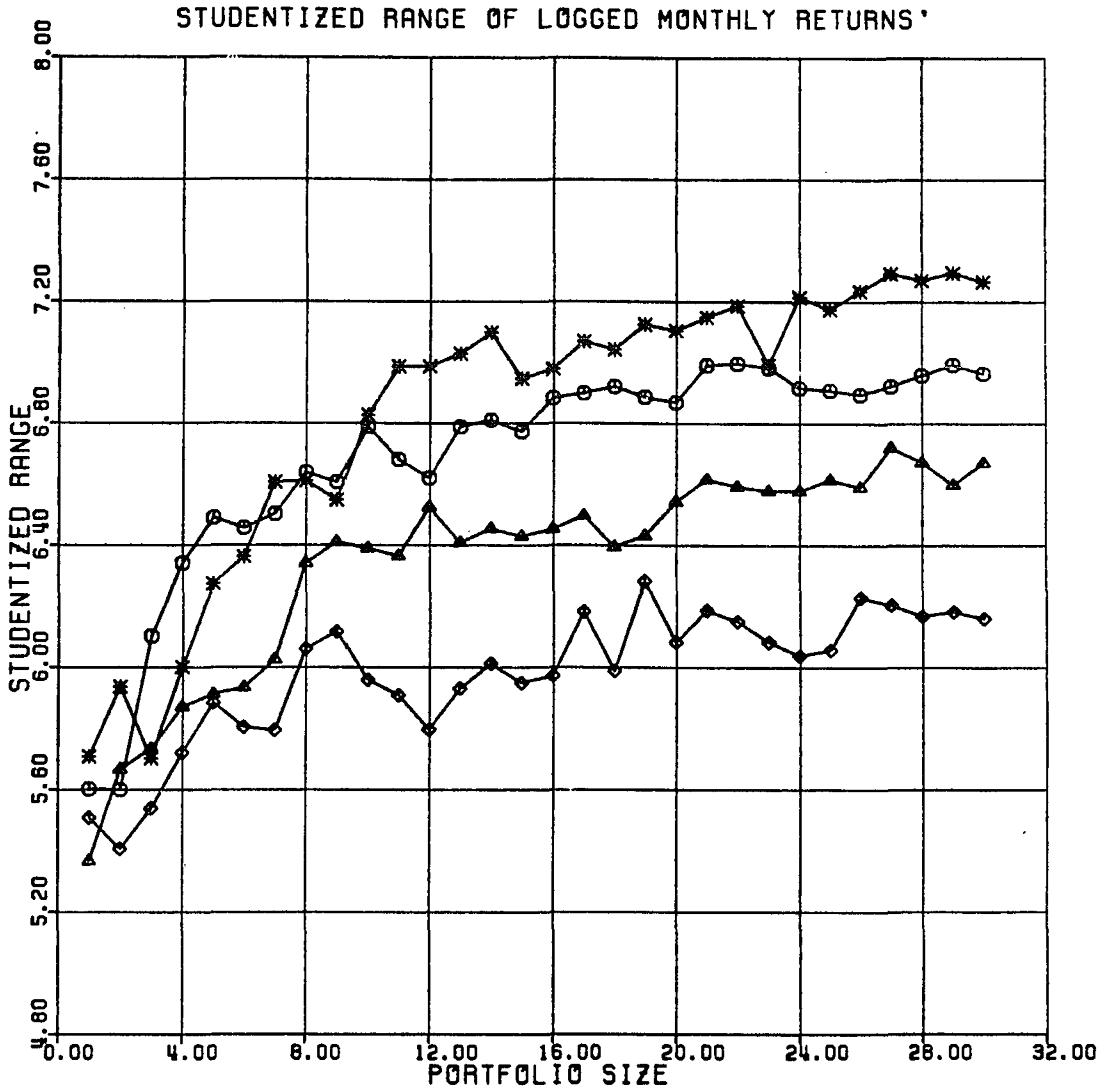
Portfolio Size	Percent Correctly Classified	Portfolio Size	Percent Correctly Classified
1	27.50	16	60.00
2	32.50	17	55.00
3	35.00	18	56.25
4	37.50	19	50.00
5	41.25	20	58.75
6	33.75	21	62.50
7	38.75	22	57.50
8	35.00	23	52.50
9	32.50	24	55.00
10	46.25	25	60.00
11	47.50	26	63.75
12	52.50	27	61.25
13	47.50	28	66.25
14	57.50	29	70.00
15	47.50	30	63.75

Figure 9



Symbol	Marketability Group
✱—✱	1 (Low)
◇—◇	2
▲—▲	3
⊖—⊖	4 (High)

Figure 10



Symbol	Marketability Group
✱—✱	1 (Low)
◇—◇	2
▲—▲	3
⊖—⊖	4 (High)

days and a few days when major news resulted in returns of relatively large magnitude. High levels of marketability, on the other hand, typically indicate high levels of investor interest resulting in very efficient information flow. As such most returns will fall close to the mean. However, when major news breaks there will be a very quick response resulting in full discounting of the news in one day, thus providing a large magnitude return for that day. Although the causes are quite different, in both cases the resulting distributions will have will be similarly shaped with high peaks at the mean and fat tails.

The discriminant analyses of the kurtosis of the monthly sample, showed large increases in classification ability as portfolio size increased. Tables 21 and 22 reveal that only about thirty percent of the small portfolios could be classified correctly. With the largest portfolios this percentage increased to over sixty percent.

An examination of the data in Figures 9 and 10 reveals that the increasing kurtosis noticed in the daily sample is still evident, but far less pronounced. The ordering of the groups is similar to what was found previously. Now, however, the four marketability levels are distinct instead of forming two groups. With the exception of the least marketable portfolios, kurtosis seems to be directly related to marketability. As previously discussed the degree of information efficiency may provide the explanation for the direct relationship. The sample with the lowest marketability seems to be segmented from this phenomenon. The most plausable explanation for this segmentation is an over abundance of no-trade and no information days. This possibility was not explored further.

Summary and Conclusions

In an effort to answer the question of the effects of marketability on return distributions, discriminant analysis was used to examine the first four moments of the return distributions. A fifth measure related to the shape of the distributions was also used in the analysis. Analysis of variance was also performed on each of these measures to verify the results of the discriminant analysis. In order to determine the impact of the holding period on the effects of marketability, both daily and monthly holding period returns were examined. The analyses were then replicated over portfolios of various sizes to provide an insight into the effect of diversification on the effects of marketability that were found for individual securities.

The tests that were performed on the sample data showed, without a doubt, that marketability does effect the shape of the return distribution of securities. The most pronounced effect was found to be on the standard deviation. The group representing the highest level of marketability was well separated from the other three, which could not be distinguished from each other. The other distributional characteristics showed weaker discriminating ability. These results are not consistent with either of the Cohen, Maier, et al. theories involving separation and homogeneity of expectations.

The daily returns sample frequently generated conflicting results. A daily holding period may be too short to capture characteristics associated with marketability, particularly low levels of marketability. When the longer holding period, one month, was investigated the results were more internally consistent. The results were even stronger when random portfolios were formed.

Again the portfolios provided strong support for the contention that marketability affects return distributions. In all cases, as portfolio size increases the differences between groups remain and are strengthened. This indicates that even for portfolios of size thirty, marketability appears to be a non-diversifiable factor.

A classic risk-return tradeoff is generally observed with respect to the mean returns and the mean standard deviation. The highest marketability group exhibited a lower return than expected based on its standard deviation, indicative of a marketability premium on price. Similarly, the lowest group provided an excess return for its risk characteristics, likely a non-marketability discount. However, marketability is not synonomous with lower risk, at least as measured by the second moment of the distribution, standard deviation. There is a strong indication of a direct relationship between standard deviation and marketability.

To a lesser extent marketability is inversely related to skewness and has a direct relationship with kurtosis. At low levels of marketability the level of kurtosis was found to increase, forming an exception to the normal pattern. As with the results of the study of individual securities, the results of the portfolio analysis are inconsistent with the Cohen, Maier, et al. theories.

A final observation about the portfolio results should be made. The full effect of diversification seems to be realized by portfolios of size twelve, indicating that marketability does not affect the speed of diversification.

Marketability appears to be a significant factor in the risk-return relationship that is realized in the market place. Not only does it effect the shape of the distribution for individual securities, but it also appears to have a non-diversifiable component that effects the shape of portfolio return distributions.

Chapter V

SUMMARY AND CONCLUSIONS

The development of any model of capital asset pricing which is to be operational as well as theoretical must address the return generation process and its observable output. A variety of studies have been undertaken in an attempt to provide a more comprehensive understanding of the market process and the resultant distribution of security returns.

These studies have produced conflicting results as to the general shape of the return distributions. They conclude that the normal distribution, as assumed for example by the Capital Asset Pricing Model, is not a good characterization of the returns observed in the market. Beyond this general conclusion, however there is little agreement. This lack of agreement is strongly indicative of the existance of one or more elements related to the shape of the observed distributions, but left out of the analyses. An increasing amount of research is being done questioning the premise that a single process can be used to describe all return distributions. That marketability may be one factor influencing this process has been mentioned in several places but has received little attention. The work that has been done involving marketability has been supportive of a relationship with the return generation process.

This dissertation, in addition to reexamining the previous hypotheses, develops a new technique for the investigation of the characteristics of distributions. The investigation also explores the effects of differences in marketability on diversification. These analyses are applied to a stratified sample of four hundred stocks drawn from the New York Stock Exchange and the American Stock Exchange and representing four levels of marketability.

Summary of the Literature

The relevant literature, as reviewed in Chapter II, can be broken down into three categories: distributional studies, marketability studies, and studies combining the two.

In the distributional studies three approaches are significant. One approach, followed by Mandelbrot, Fama and others, considered the symetric Stable Paretian distribution as a description of the empirical returns. This distribution, in general, was found inadequate in that each test produced different results. Many of the studies also found a violation of the stability property of that distribution. Others, including that of Officer, found violations of the assumption that the second and higher moments are undefined for the Stable Paretian distribution. These studies also reported some indication of skewness.

The second approach, followed by Press, provided the only investigation of a compound events model. The study contained limited empirical testing because of problems in parameter estimation. In those cases that the parameter estimation procedures were successful, his ability to model the observed price changes was quite good. The validity of such a model for describing the return patterns observed in the market place was neither confirmed nor rejected, but left for further investigation.

The third approach to describing the empirical return distributions, followed by Paretz, and by Blattberg and Gonedes, again relied on a single statistical distribution, the scaled Student t. Since this is a sampling distribution, as opposed to a limiting distribution, its shape and the number of defined moments are not rigid. Both elements are captured in the single "degrees of freedom" parameter. In both studies this distribution provided a better fit to the real world than the Stable Paretian or a compound events model. The "estimated" parameter was consistantly found to be four or larger, indicating that the first four moments were defined and finite; and the parameter was not the same for all of the series that were tested, evidence that not all of the return distributions came from a single generating process. The wide variety of results suggests that various factors influence the return generating process. The remainder of the relevant literature considers marketability as one such factor.

In establishing the relationship between marketability and returns, three articles were reviewed. Tinic, and Tinic and West examined the effect of thinness on dealers' and specialists' bid-ask spreads, finding a strong relationship. They also examined the effect of thinness on the variability of bid-ask spreads and found no relationship. The third article, by Ying, found a high degree of correlation between trading volume and price changes, again supportive of a relationship between marketability and the return generating process.

Finally, several papers were reviewed that directly examined the relationship between marketability and return distributions. Cohen, Maier, et al. theorized that if expectations are homogeneous and investors hold securities in proportion to the securities' market value, then marketability would have no effect on the distributions. If neither condition is true an increase in the standard deviation should be found for thinly traded issues (an inverse relationship with respect to marketability), but the mean should be uneffected.

Cohen, Ness, et al., in a separate work, found results generally supportive of the second case of the theory. Criticisms of their methodology drastically reduced the credibility of the results, however, and again left the area in a state of conjecture. One additional study by Senchack and Barnett indicated that thin markets tended to produce an increase in variability, positive skewness, and leptokurtosis.

These studies and others reviewed in Chapter II provide the justification for continued examination of the relationship between market return distributions and marketability by demonstrating that the shape of return distributions is open to question and that marketability is related to market returns.

Procedure

Chapter III describes the selection of the stratified sample of four hundred securities and the development of the techniques of analysis. Marketability was defined as the ratio of shares traded to shares outstanding in 1978. There were four strata in the sample, each representing a different degree of marketability and each containing one hundred stocks listed on the New York or American Stock Exchange. For each security two time series of returns were collected, one series containing 252 daily returns covering one year beginning on January 2, 1978, and the other containing 60 monthly returns covering a five-year period ending June, 1979.

Five statistical measures were designated to characterize the empirical return distributions for each security. These were the mean, standard deviation, skewness, kurtosis, and studentized range. The best available technique for analysis was determined to be multiple discriminant analysis. The discussion also established the form of the distance function and how this function is to classify observations into the four strata.

The problem of non-normal variable inputs to the discriminant function was addressed in relation to an appropriate interpretation of the empirical results. Much of the analysis must be done on a relative basis since normal parametric testing could not be used.

A procedure for the construction of portfolios within each strata was developed to investigate the systematic portion of differences in the return distributions. For each strata, twenty portfolios for each of the portfolio sizes of one through thirty securities were formed. Discriminate analysis applied to these portfolios would give an insight into those differences in return distributions associated with marketability that are non-diversifiable.

The chapter served as a prelude to understanding the results of the empirical testing examined in the subsequent chapter.

Major Results

Examination of the market's response to differences in marketability as revealed in the price-earnings ratios was conducted first. That examination revealed that the market did perceive differences in marketability and responded by rewarding the high marketability group relative to the other three groups. Price-earnings ratios were not distributed uniformly across the entire sample, nor were they strongly directly related across the four groups.

Both the daily and monthly samples were subjected to two four factor discriminant tests, one based on the first four moments, the other replacing the fourth moment with the studentized range. In all four tests the correct classification was slightly over forty percent of the samples. This is not a strong classification ability, but is significantly above the twenty-five percent expected from random assignment. A conclusion that marketability does effect the characteristics of the return distributions was sufficiently supported to indicate a need to examine each characteristic separately. When this was done, the standard deviation produced the most significant differences. Based on the daily return sample a correct classification rate of 35.5 percent was achieved. An examination of the mean levels of the standard deviation for each group produced an unexpected a U-shaped pattern as marketability increased. The monthly return sample did not repeat the pattern, instead the standard deviation increased continuously as marketability increased.

The tests on the mean and skewness had less significant results. Simkowitz and Beedles' results were confirmed with respect to the diversifiability of skewness when the high marketability group was examined. The low marketability group, however exibited a direct relationship between skewness and portfolio size. The two measures of kurtosis yielded statistically significant results, but were not impressive with respect to their classification ability.

These results appear contrary to the generally accepted theory of investor behavior that an investor requires an increase in return as an inducement to accept additional standard deviation (risk). These findings, however, are for ex post returns on individual securities, and thus include nonsystematic sources of variance. If investors price assets based upon that asset's contribution to a portfolio, the results for the portfolios should reveal more of the influence of marketability. Results for the portfolios were consistent with the traditional risk-return trade-off.

Within each stratum of marketability, portfolios of size from 1 to 30 securities were randomly formed to reduce the nonsystematic effects. The discriminating ability of the four factor models increased dramatically for both holding periods when the larger portfolios were examined. All four of the models produced classification abilities far in excess of ninety percent providing strong evidence of the existance of a link between marketability and the return generating process and that this link is not eliminated through diversification.

When each characteristic was examined separately, several other relationships appeared. In general the results for portfolios constructed from the daily return sample revealed odd pairings and orderings of the mean levels for each characteristic. These results may reflect no-trade days and price concessions in the lower marketability groups. These results were not observed in the results of the monthly return sample, raising the question of the applicability of daily holding periods in most studies of the effects of marketability.

The examination of the distributions' means for the monthly return sample found the classification ability improved significantly with larger portfolios. The plot of the mean levels showed a consistant ordering for portfolios with at least twelve stocks. Although there was a tendency for a direct relationship to exist the highest marketability group showed a lower return, while the lowest marketability group had a mean return well above what would have been expected. Apparently a premium and a discount, respectively, were being assessed in response to high and low levels of marketability.

The standard deviation was again found to be the major contributer to the effectiveness of the four factor discriminant models. A direct relationship was found between standard deviation and marketability. This ordering is consistent, under a classic risk-return trade-off, with that observed for the means, but inconsistent with the intuitive concept that marketability reduces risk.

Skewness was not as good as the standard deviation in terms of discriminating, but did reveal a very consistant inverse relationship to marketability.

Like the skewness coefficient, the two measures of kurtosis revealed for the larger portfolios a reasonable ability to separate the marketability groups and a general direct relationship to marketability. The one exception to the relationship was the lowest marketability group had the highest level of kurtosis. It was speculated that this exception may have been the result of a multitude of no-trade days.

All of the forgoing relationships indicated that marketability has a non-diversifiable component. In several cases the strength of the relationship actually increased for the larger portfolios. Marketability was found to not effect the speed of diversification. In most instances all obtainable, diversification had been achieved where portfolio size reached twelve.

Conclusions and Implications

The usual caveats hold. The conclusions of this study are applicable only for stocks listed on the New York and American Exchanges during this particular interval of time. Moreover, these conclusions are valid only for holding periods of one day or month. While these conclusions may be applicable beyond these boundaries, additional empirical study would be necessary to support such claims.

This study has provided additional evidence against the acceptance of the Stable Paretian distribution as a model of market return distributions. The higher order moments, second and above, were quite well behaved, which is a direct contradiction of the assumptions of the Stable Paretian distribution. To the extent that the well behaved moments are in agreement with the assumptions of the scaled Student t distribution, that distribution is supported. Paretz and Blattberg and Gonedes had found that the best fit scaled Student t distributions consistantly had a "degrees of freedom" parameter sufficient for at least the first four moments to be well behaved. This agreement of results is not an argument for the use of the scaled Student t distribution as a model of market returns, however, but should instead be considered as not contradicting such a model.

The theoretical constructs of Cohen, Maier, et al. are not supported by this study. They contended that the mean returns should be independent of marketability. While individual securities showed no relationship, the portfolios revealed increases in mean returns as marketability increases. Cohen, Maier, et al.'s contentions with regard to the standard deviation were also contradicted. Their theory suggests that either an inverse relationship or independence should exist, depending on the assumptions made. This study, instead, found a direct relationship in both the security and portfolio analyses.

The inverse relationship between marketability and both skewness and kurtosis that was found here confirms the results of the study done by Senchack and Barnett. Their finding of an inverse relation to the standard deviation was contradicted by this study, however.

The results of this study are inconsistent with the simplified pricing model based only upon a single market index factor. Return distributions were not found to be normally distributed, and were found to respond to another factor, marketability. Whatever other factors may influence security returns is left for other researchers to determine.

The influence of marketability on returns is important not only for the speculator interested in getting in and out quickly, but also for the investor with longer holding periods. For portfolios, it was shown that skewness and kurtosis have non-diversifiable components and that their interaction with marketability is enhanced, not diminished, as the number of securities is increased. The relationship between marketability and the diversifiability of skewness also implies that marketability must be considered in building portfolios.

The fact that marketability was found to effect the distribution of ex post returns also has implications with regard to the effectivness of the specialist. One of his duties is to provide liquidity to the market. The results of this study do not allow any conclusions to be drawn about the absolute amount of liquidity that is created by the specialist. They do, however, indicate that whatever amount is created does not eliminate significant differences in return distributions associated with differences in marketability.

This study has examined the relationship between marketability and the generation of returns in the market place. Only recently have the complexities of the generation process begun to be understood. Each aspect of this understanding has been the result of many researchers making a small contribution that could be verified, built upon, and combined with other work. This study, also, marks a small contribution that may lead to a fuller understanding of the market mechanism.

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Appendix A

STABLE PARETIAN DISTRIBUTION

This appendix is designed to acquaint the reader with the properties of the stable Paretian distribution, as it is used in the modeling of return distributions. The material contained here is not essential to understanding the arguments presented in the main body of this work. Its intent is to provide a more complete discussion of the stable Paretian distribution for those who are interested. Excellent discussions of this distribution are found in Gnedenko and Kolmogorov and Press.[1]

The stable Parentian distribution is a seldom used distribution, primarily because of a lack of knowledge about its properties. In general, the density function is not determined, necessitating the distribution be defined in terms of its characteristic function. The log characteristic function for the symmetric case is:

$$\ln \phi_x(t) = i\delta t - |ct|^{\alpha}$$

where t is some real number, δ is a location parameter, $c > 0$ is a scale parameter, $i = \sqrt{-1}$, and the characteristic exponent is $\alpha \in (0,2)$.

As this is really a family of distributions, its properties are highly dependent on the value given its parameters. The most important

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[1]B. V. Gnedenko and A. N. Kolmogorov, Limit Distributions for Sums of Independent Random Variables, trans. K. L. Chung (Reading, Mass.: Addison-Wesley, 1954), Chap. 7; and S. J. Press, Applied Multivariate Analysis (New York:Holt, Rinehart & Winston, 1972), Chap. 6.

of these is the characteristic exponent. When $\alpha < 1$ all moments are undefined and appear infinite. At $\alpha = 1$ it is a Cauchy distribution. For values of α between one and two the mean is defined, but no other moments are. An α value of two produces a normal distribution.

Another important property of these distributions is that they are stable under addition. By this it is meant that the sum of any number of independent random variables with characteristic exponent α^* will also be distributed with a characteristic exponent α^*.

If $\alpha < 2$, the distribution will have fatter tails and a higher peak at its centroid than a normal distribution. This property becomes more pronounced with smaller values of α.

Appendix B

SCALED STUDENT t DISTRIBUTION

This appendix is intended to provide a fairly rigorous development of the scaled Student t distribution as it is applied as a modeling distribution, and contains information about the characteristics of the distribution. The approach used will closely follow the one used by Paretz.[1] It should be emphasized that the information contained in this appendix is intended for the interested reader and is not essential to the understanding of the main text.

Let us begin by considering the Brownian motion model developed by Osborne.[2]

$$f(y) = \frac{\exp(-y^2/2\sigma^2\tau)}{(2\pi\sigma^2\tau)^{1/2}} \tag{1}$$

where $y = \ln[p(t) \pm \tau)/p(t)]$, which represents the continuously compounded return over time period τ, and σ is the variance of y.

This model is based on the assumption of a constant variance for y. If this is not true, but instead the variance of y is itself a random variable, the distribution given in (1) is a conditional distribution.

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[1]Peter D. Paretz, "The Distribution of Share Price Changes," Journal of Business, 45:49-52, January 1972.

[2]M. Osborne, "Brownian Motion in the Stock Market," Operations Research 7:145-173, March-April 1959.

Before restating (1) as a conditional distribution let us also make two additional changes. First, let us include the possibility of a non-zero mean, μ. Also let us define $\tau = 1$ (an arbitrary unit time interval). The results can be expressed as follows:

$$f(y|\sigma^2) = \frac{\exp[-(y - \mu)^2/2\sigma^2]}{(2\mu\sigma^2)^{\frac{1}{2}}} \tag{2}$$

In order to develop a distribution for y which is observable, it will be necessary to assign a distribution to σ^2. The distribution chosen for this purpose is an inverted gamma. A priori, this choice is somewhat arbitrary. It was made largely due to the fact that it is statistically well understood, is properly behaved (i.e., strictly non-negative), and will produce a result of the type desired. It can also be thought of in a Bayesian sense as an assigned prior distribution for an unknown parameter. The inverted gamma distribution can be expressed as

$$g(\sigma^2) = \sigma_0^{2m}(m - 1)^m \sigma^{-2(m+1)} e^{-(m-1)\sigma_0^2/\sigma^2} / \Gamma(m) \tag{3}$$

where $\sigma_0^2 = E(\sigma^2)$ and the variance of σ^2 is $\sigma_0^4/(m-2)$.

If we let h(y) represent the unconditional distribution of y, it is clear that

$$h(y) = \int_0^\infty f(y|\sigma^2)g(\sigma^2)d\sigma^2 \tag{4}$$

Substituting $g(\sigma^2)$ from (3) into (4) and integrating we find

$$h(y) = [1 + (y - \mu)^2/\sigma_0^2(2m - 2)]^{-m-\frac{1}{2}}\Gamma(m)[(2m - 2)\pi]^{\frac{1}{2}}\sigma_0 \tag{5}$$

This is a well know expression of a t distribution with 2m degrees of freedom, with the exception of a scale factor of $[2m/(2m - 2)]^{\frac{1}{2}}$.

The above mentioned scale factor leaves intact the characteristic fat tails of a t distribution, however, it generates a high peaked center. The extent of these two characteristics is determined by the value assigned to the degrees of freedom parameter 2m. Larger values of the parameter result in distributions closer in form to those of a normal.

One final point should be made with regard to the scaled t distribution. Although the distribution can be fully defined by the one parameter, the degrees of freedom, it is not the only valid description. In particular, the moments of this distribution exist and are finite for all moments of order less than the degrees of freedom parameter.

Appendix C

LISTING OF FIRM SAMPLE BY MARKETABILITY GROUP

Group 1: Low Marketability

Company Name	Exchange	Percent of Outstanding Shares Traded Per Year
American Brands, Inc.	NYSE	10.753
Aro Corp.	NYSE	9.325
Atlantic City Electric Co.	NYSE	10.581
Ball Corp.	NYSE	8.889
Barclay Industrials, Inc.	AMEX	9.524
Belding Heminway, Inc.	NYSE	9.604
Brooklyn Union Gas Co.	NYSE	9.718
Brown Group, Inc.	NYSE	11.472
Carnation Co.	MAEX	9.793
Castle & Cooke, Inc.	NYSE	9.996
Ceco Corp.	NYSE	11.443
Central Illinois Light Co.	NYSE	11.287
Central Maine Power Co.	NYSE	9.715
Chadwick Miller, Inc.	AMEX	10.179
Commercial Metals Co.	AMEX	9.597
Community Public Service Co.	NYSE	11.367
Conrock Co.	AMEX	9.431
Continental Materials Corp.	AMEX	9.873
Credithrift Financial, Inc.	NYSE	11.001
Crompton, Inc.	AMEX	9.200
Culbro Corp.	NYSE	10.668
Dial Corp.	NYSE	8.640
Duquesne Light Co.	NYSE	10.756
Eagle Picher Industries, Inc.	NYSE	10.096
Edison Brothers Stores, Inc.	NYSE	10.371
Empire District Electric Co.	NYSE	10.576
Equitable Gas Co.	NYSE	9.436
First Virginia Banks, Inc.	NYSE	8.562
Franks Nursery Sales, Inc.	AMEX	11.484
Getty Oil Co.	NYSE	10.087
Glatfelter (P. H.) Co.	AMEX	9.808
Grand Auto, Inc.	AMEX	10.844
Graniteville Co.	NYSE	10.850
Gross Telecasting, Inc.	AMEX	8.899
Hampton Industries, Inc.	AMEX	10.254

Group 1: Low Marketability (continued)

Company Name	Exchange	Percent of Outstanding Shares Traded Per Year
Hershey Foods Corp.	NYSE	9.259
Hillengrand Industries, Inc.	NYSE	9.427
Hobard Corp.	NYSE	10.842
Indiana Gas, Inc.	NYSE	11.138
Interstate Power Co.	NYSE	10.026
Jefferson Pilot Corp.	NYSE	11.289
Jorgensen (Earle M.) Co. (Delaware)	NYSE	8.760
Kay Corp.	AMEX	8.795
Kennametal, Inc.	NYSE	11.041
Key Co.	AMEX	10.996
King Radio Corp.	AMEX	10.980
Lodge and Shipley Co.	AMEX	9.491
Louisville Gas and Electric Co.	NYSE	10.725
Macrodyne Industries, Inc.	AMEX	10.184
McIntyre Mines, Ltd.	NYSE	9.647
Mercantile Stores, Inc.	NYSE	9.528
Mic Continent Telephone Corp.	NYSE	8.679
National Fuel Gas Co. (New Jersey)	NYSE	9.307
National Service Industries, Inc.	NYSE	9.718
Newcor, Inc.	AMEX	11.223
Niagara Frontier Services, Inc.	AMEX	11.396
Nicor, Inc.	NYSE	10.466
North American Philips, Corp.	NYSE	8.996
Northgate Exploration, Ltd.	NYSE	10.152
Ohio Sealy Mattress Manufacturing Co.	AMEX	8.917
Orange and Rockland Utilities, Inc.	NYSE	10.207
Pentron Industries, Inc.	AMEX	9.072
Phoenix Steel Corp.	AMEX	10.344
Piedmont Natural Gas, Inc.	NYSE	9.012
Potlatch Corp.	NYSE	11.487
Prentice-Hall, Inc.	AMEX	9.315
Procter and Gamble Co.	NYSE	10.260
Rochester Gas & Electric Corp.	NYSE	9.677
Sav On Drugs, Inc.	NYSE	10.570
Schenuit Industries, Inc.	AMEX	10.668
Sealed Power Corp.	NYSE	11.409
Sears Roebuck & Co.	NYSE	11.354
South Jersey Industries, Inc.	NYSE	10.200
Southern Indiana Gas and Electric Co.	NYSE	10.463
S. S. P. Industries	AMEX	10.604
Standard Oil Co. (Indiana)	NYSE	10.432
Standard Products Co.	AMEX	10.413
Standex International Corp.	NYSE	8.877

Group 1: Low Marketability (continued)

Company Name	Exchange	Percent of Outstanding Shares Traded Per Year
Starrett Housing Corp.	AMEX	10.949
Steward Warner Corp.	NYSE	10.373
Superior Industries International, Inc.	AMEX	10.766
Technicon Corp.	NYSE	8.733
Thrifty Corp.	NYSE	10.338
Toledo Edison Co.	NYSE	11.448
Tootsie Roll Industries, Inc.	NYSE	9.229
Unarco Industries, Inc. (Delaware)	NYSE	
United Illum Co.	NYSE	9.936
United Jersey Banks (Hackensack)	NYSE	8.760
Vertipile, Inc.	AMEX	8.767
V. S. I. Corp.	NYSE	8.594
Washington Gas and Light Co.	NYSE	9.464
Washington Water and Power Co.	NYSE	9.913
Wean United, Inc.	NYSE	10.113

Group 2: Lower Middle Marketability

Company Name	Exchange	Percent of Outstanding Shares Traded Per Year
Alcolac, Inc.	AMEX	19.418
Allied Products Corp. (Delaware)	NYSE	17.964
American Cyanamid Co.	NYSE	19.153
Amster Corp.	NYSE	17.935
Associated Dry Goods Corp.	NYSE	18.562
A. Z. L. Resources, Inc.	AMEX	19.223
Bayuk Cigars, Inc.	NYSE	19.241
Bendix Corp.	NYSE	17.930
Big Three Industries, Inc.	NYSE	19.093
Caldor, Inc.	AMEX	18.869
Central Telephone & Utilities Corp.	NYSE	19.088
Certain Teed Corp.	NYSE	17.786
Charter Co.	NYSE	18.379
Charter (New York) Corp.	NYSE	17.924
Chelsea Industries, Inc.	NYSE	18.249
Cohu, Inc.	AMEX	19.348
Colgate Palmolive Co.	NYSE	18.029
Compudyne Corp.	AMEX	18.283
Cone Mills Corp.	NYSE	19.307
Continental Corp.	NYSE	17.885
Continental Group, Inc.	NYSE	18.202
C. P. C. International, Inc.	NYSE	19.557
Crown Zellerbach Corp.	NYSE	18.809
Cyprus Mines Corp.	NYSE	18.910
Dennison Manufacturing Co.	NYSE	18.613
Dover Corp.	NYSE	18.663
Du Pont (E. I.) De Nemours & Co.	NYSE	19.568
Duke Power Co.	NYSE	18.618
Dutch Boy, Inc.	NYSE	18.081
Electro Audio Dynamics, Inc.	AMEX	19.110
Electrographic Corp.	AMEX	18.511
Engelhard Minerals & Chemicals	NYSE	18.707
Ethyl Corp.	NYSE	17.806
Federal Paper Board, Inc.	NYSE	19.196
Federal Mogul Corp.	NYSE	18.456
Federated Deaprtment Stores, Inc.	NYSE	18.012
Firestone Tire & Rubber Co.	NYSE	18.169
General Public Utilities Corp.	NYSE	17.942
Gleason Works	NYSE	17.836
Gulf United Corp.	NYSE	19.064
Hart Schaffner & Marx	NYSE	18.909
Household Financial Corp.	NYSE	19.347
Houston Natural Gas Corp.	NYSE	18.820
I. N. A. Corp.	NYSE	18.012
Inco Ltd.	NYSE	18.452

Group 2: Lower Middle Marketability (continued)

Company Name	Exchange	Percent of Outstanding Shares Traded Per Year
Interpace Corp.	NYSE	17.954
Jantzen, Inc.	NYSE	18.484
Jewelcor, Inc.	NYSE	19.493
Keystone Consolidated Industries, Inc.	NYSE	19.494
Kidde (Walter) & Co., Inc.	NYSE	19.142
Learonal, Inc.	AMEX	18.107
Litton Industries, Inc.	NYSE	19.043
Lubrizol Corp.	NYSE	18.405
Marlene Industries Corp.	AMEX	19.433
McLean Trucking Co.	NYSE	18.034
McLouth Steel Corp.	NYSE	19.050
Merck & Co., Inc.	NYSE	18.923
Mirro Aluminum Co.	NYSE	19.253
Mony Mortgage Investments	NYSE	18.401
Morgan (J. P.) & Co., Inc.	NYSE	18.405
North American Coal Corp.	NYSE	18.642
Northern States Power Co., Minnesota	NYSE	18.602
Northwestern Mutual Life, Mortgage and Realty	NYSE	19.000
N. V. F. Co.	NYSE	17.952
O. K. C. Corp.	NYSE	18.352
Outboard Marine Corp.	NYSE	19.450
Pfizer, Inc.	NYSE	18.260
Phillips Petroleum Co.	NYSE	18.463
Proler International Corp.	NYSE	18.360
Public Service Co. (New Mexico)	NYSE	19.434
Reynolds (R. J.) Industries, Inc.	NYSE	18.829
Riblet Products Corp.	AMEX	18.165
Rohm & Haas Co.	NYSE	18.178
Rollins, Inc.	NYSE	18.423
Russell, Inc.	AMEX	19.047
Salant Corp.	NYSE	18.487
Sigma Instruments, Inc.	AMEX	19.519
South Carolina Electric and Gas Co.	NYSE	18.130
Southern Union Co.	NYSE	18.412
Southland Royalty Co.	NYSE	17.872
St. Joe Minerals Corp.	NYSE	17.961
St. Regis Paper Co.	NYSE	
Standard Brands, Inc.	NYSE	19.467
Supermarkets General Corp.	NYSE	18.993
Teradyne, Inc.	NYSE	19.227
U. G. I. Corp.	NYSE	17.922
Union Communication Corp.	NYSE	18.349
Union Oil Company of California	NYSE	19.089

Group 2: Lower Middle Marketability (continued)

Company Name	Exchange	Percent of Outstanding Shares Traded Per Year
United Telecommunications, Inc.	NYSE	18.137
Universal Leaf Tobacco, Inc.	NYSE	19.242
Virginia Electric & Power Co.	NYSE	19.343
Wang Laboratories, Inc.	AMEX	19.403
Westvaco Corp.	NYSE	18.559

Group 3: Upper Middle Marketability

Company Name	Exchange	Percent of Outstanding Shares Traded Per Year
Aegis Corp.	AMEX	24.307
Alcan Aluminum Ltd.	NYSE	26.074
Alpha Industries, Inc.	AMEX	26.606
Alpha Portland Industries, Inc.	NYSE	26.050
Amcord, Inc.	NYSE	25.938
American Bakeries Co.	NYSE	26.167
American District Telegraph Co.	NYSE	25.927
American Hospital Supply Corp.	NYSE	25.021
Anthony Industries, Inc.	AMEX	24.335
Bache Group, Inc.	NYSE	26.529
Bankers Trust New York Corp.	NYSE	26.631
Barnes Engineering Co.	AMEX	26.276
Binney & Smith, Inc.	NYSE	26.617
Brockway Glass Co.	NYSE	25.430
Campbell Red Lake Mines, Ltd.	NYSE	25.181
Carolina Power & Light Co.	NYSE	24.067
Central & Southwest Corp.	NYSE	25.334
Cenvill Communities, Inc.	AMEX	25.176
Cities Service Co.	NYSE	24.726
Coleman, Inc.	AMEX	24.755
Connecticut General Mortgage and Realty Investments	NYSE	26.278
Consolidated Freightways, Inc.	NYSE	26.176
Cooper Industries, Inc.	NYSE	25.679
Corning Glass Works	NYSE	26.796
Echlin Manufacturing Co.	NYSE	25.684
Esmark, Inc.	NYSE	24.055
Fabri-Centers of America, Inc.	NYSE	24.691
Fieldcrest Mills, Inc.	NYSE	26.403
Foremost McKesson, Inc.	NYSE	24.239
Freeport Minerals Co.	NYSE	25.787
G. A. F. Corp.	NYSE	24.398
General Foods Corp.	NYSE	25.444
Genisco Technology Corp.	AMEX	25.498
Goodyear Tire & Rubber Co.	NYSE	24.307
Gray Drug Stores, Inc.	NYSE	26.054
Great American Industries, Inc.	AMEX	24.235
Harcourt, Brace, Jonanovich, Inc.	NYSE	26.088
Heileman (G.) Brewing, Inc.	NYSE	24.437
Heinnicke Instruments Co.	AMEX	24.528
Hipotronics, Inc.	AMEX	26.189
Horn & Hardart Co.	AMEX	24.855
I. E. Industries, Inc.	NYSE	25.105
Idaho Power Co.	NYSE	26.068
Ingersoil Rand Co.	NYSE	25.338
International Proteins Corp.	AMEX	26.114

Group 3: Upper Middle Marketability (continued)

Company Name	Exchange	Percent of Outstanding Shares Traded Per Year
Jostens, Inc.	NYSE	26.419
Kansas City Southern Industries, Inc.	NYSE	25.704
Kansas Nebraska Natural Gas, Inc.	NYSE	25.278
Keene Corp.	NYSE	24.904
Keller Industries, Inc.	NYSE	26.959
Kimberly Clark Corp.	NYSE	24.397
Kings Department Stores, Inc.	NYSE	24.089
Kroger Co.	NYSE	25.759
Libbey-Owens-Ford Co.	NYSE	24.363
Marion Labs, Inc.	NYSE	24.154
May Department Stores	NYSE	26.566
M. C, A., Inc.	NYSE	25.874
McGraw Hill, Inc.	NYSE	25.921
Mead Corp.	NYSE	24.738
Melville Corp.	NYSE	24.383
Middle South Utilities, Inc.	NYSE	25.667
Monarch Machine Tool Co.	NYSE	25.564
National Can Corp.	NYSE	26.435
New Hampshire Ball Bearing	AMEX	26.475
Ogden Corp.	NYSE	25.457
Oklahoma Natural Gas Co.	NYSE	25.082
Oneida Ltd.	NYSE	25.677
Panhandle Eastern Pipe Line Co.	NYSE	26.937
Parsons Corp.	AMEX	25.849
Pennzoil Co.	NYSE	24.817
Pillsbury Co.	NYSE	25.751
Pittway Corp.	AMEX	26.084
Products Research & Chemical Corp.	NYSE	26.733
Pullman, Inc.	NYSE	26.852
Purolator, Inc.	NYSE	26.770
R. B. Industries, Inc.	AMEX	25.752
Republic Steel Corp.	NYSE	24.838
Rex Noreco, Inc.	AMEX	24.221
Safeway Stores, Inc.	NYSE	24.205
Santa Fe Industries, Inc.	NYSE	25.721
Scott Foresman & Co. (Delaware)	NYSE	25.568
Seagrave Corp.	NYSE	25.355
St. Louis San Francisco Railway Co.	NYSE	25.734
Stevens (J. P.) & Co. Inc.	NYSE	25.362
Storer Braodcasting Co.	NYSE	25.500
Stride Rite Corp.	NYSE	25.611
Suave Shore Corp.	NYSE	25.895
Superior Oil Co.	NYSE	24.180
Texas Eastern Corp.	NYSE	24.526

Group 3: Upper Middle Marketability (continued)

Company Name	Exchange	Percent of Outstanding Shares Traded Per Year
Texas Gas Transmission Corp.	NYSE	24.236
Texas Utilities Co.	NYSE	26.541
T. R. W., Inc.	NYSE	26.923

Group 4: High Marketability

Company Name	Exchange	Percent of Outstanding Shares Traded Per Year
American Seating Co.	AMEX	44.667
Arctic Enterprises, Inc.	NYSE	46.206
Arvin Industries, Inc.	NYSE	51.300
Asarco, Inc.	NYSE	43.239
Avnet, Inc.	NYSE	42.976
Bancal Tri State Corp.	NYSE	43.004
Banner Industries, Inc.	NYSE	47.136
Bard (C. R.), Inc.	NYSE	50.874
Berkey Photo, Inc.	NYSE	41.794
Bethlehem Steel Corp	NYSE	48.124
Blue Bell, Inc.	NYSE	42.503
Bradford National Corp.	AMEX	47.065
Braniff International Corp.	NYSE	42.683
Browning Ferris Industries, Inc.	NYSE	47.203
Buffalo Forge Co.	NYSE	42.379
Burlington Northern, Inc.	NYSE	45.367
Centex Corp.	NYSE	41.265
Century Factors, Inc.	AMEX	46.246
City Investing Co.	NYSE	48.077
Coldwell Banker & Co.	NYSE	47.492
Collins Food International, Inc.	NYSE	45.262
Community Psychiatric Centers	NYSE	48.738
Computer Sciences Corp.	NYSE	42.851
Congoleum Corp.	NYSE	44.465
Consolidated Oil & Gas, Inc.	AMEX	43.933
Crystal Oil Co.	AMEX	48.151
Disney (Walt) Productions, Inc.	NYSE	46.467
D. W. G. Corp.	AMEX	49.588
E-Systems, Inc.	NYSE	47.438
E. G. & G., Inc.	NYSE	49.283
Electronic Memories & Magnetics	NYSE	45.958
Essex Chemicals Corp.	NYSE	45.446
Evans Products Co.	NYSE	41.838
G. R. I. Corp.	AMEX	49.478
Gamble-Skogmo, Inc.	NYSE	48.574
General Portland, Inc.	NYSE	41.197
Goodrich (B. F.) Co.	NYSE	44.238
Gulton Industries, Inc.	NYSE	51.184
Harnischfeger Corp.	NYSE	44.246
Hazeltine Corp.	NYSE	41.342
Hecks, Inc.	NYSE	41.122
High Voltage Engineering Corp.	NYSE	42.052
Homestake Mining Co.	NYSE	45.549
Huyck Corp.	NYSE	41.101
Ideal Toy Corp.	NYSE	44.951

Group 4: High Marketability (continued)

Company Name	Exchange	Percent of Outstanding Shares Traded Per Year
Imperial Corporation of America	NYSE	48.241
Instruemnt Systems Corp.	AMEX	44.862
Ipco Hospital Supply Corp.	NYSE	46.277
Kaneb Services, Inc.	NYSE	43.426
Kirsch Co.	NYSE	43.002
Lear Siegler, Inc.	NYSE	42.592
Lionel Corp	NYSE	49.340
Marley Co.	NYSE	50.539
Martin Processing, Inc.	AMEX	41.571
Mego International, Inc.	AMEX	46.737
M. G. I. C. Investment Co	NYSE	44.736
Milton (Roy) Co.	NYSE	43.045
National Medical Enterprises, Inc.	NYSE	49.556
Northwest Industries, Inc.	NYSE	44.796
Omark Industreis, Inc.	NYSE	41.225
Pargas, Inc.	NYSE	46.281
Peabody International Corp.	NYSE	41.252
Phelps Dodge Corp.	NYSE	46.433
Pitney Bowes, Inc.	NYSE	48.111
Planning Research Corp.	NYSE	51.648
Portec, Inc.	NYSE	49.922
P. S. A., Inc.	NYSE	47.667
Ranchers Exploration Development Corp.	AMEX	48.826
Ranger Oil Canada Ltd.	AMEX	44.953
Republic Corp.	NYSE	44.357
Revco (D. S.), Inc.	NYSE	45.280
Rio Grande Industries, Inc.	NYSE	43.281
Robertshaw Controls Co.	NYSE	41.076
Russ Togs, Inc.	NYSE	47.230
S. C. A. Services, Inc.	NYSE	50.897
Scot Lad Foods, Inc.	NYSE	42.853
Shapell Industries, Inc.	NYSE	48.158
Sierracin Corp.	AMEX	44.569
Skyline Corp.	NYSE	41.434
Soundesign Corp.	AMEX	47.511
Standard-Pacific Corp.	NYSE	48.249
Studebaker-Worthington, Inc.	NYSE	48.727
Sun Electric Corp.	NYSE	41.569
Tappan Co.	NYSE	44.195
Teleprompter Corp.	NYSE	47.453
Total Petroleum (North American) Ltd.	AMEX	46.390
Tracor, Inc.	NYSE	50.491
United States Leasing International, Inc.	NYSE	45.153

Group 4: High Marketability (continued)

Company Name	Exchange	Percent of Outstanding Shares Traded Per Year
United States Shoe Corp.	NYSE	41.470
United Financial Corp. (California)	NYSE	44.436
Unitrode Corp	NYSE	50.517
Watkins-Johnson Co.	NYSE	43.617
Wilshire Oil Co. (Texas)	NYSE	44.492
Zayre Corp.	NYSE	45.482

VITA

Bruce Loren McManis was born April 29, 1953 in Hartford City, Indiana to Donald L. and Eileen G. McManis. He was graduated from Lincoln East High School, Lincoln, Nebraska, in 1971. Immediately following graduation he enrolled at Kansas State University and graduated in the spring of 1974 with a B.S. degree in Business Administration.

In the summer of 1974 he began an extensive tenure at Louisiana State University. Initially, he pursued and received an M.S. degree in Quantitative Methods. This degree was granted in the spring of 1976. Since then his efforts have been directed toward a Ph.D. degree with a major in Finance and a minor in Quantitative Methods. He is scheduled to receive this degree in the summer of 1981.

In the fall of 1978, Mr. McManis accepted his current position of Assistant Professor at Nicholls State University, Thibodaux, Louisiana.

He is married to the former Nancy Jenkins of Salina, Kansas.

EXAMINATION AND THESIS REPORT

Candidate: Bruce Loren McManis

Major Field: Finance

Title of Thesis: A Study of the Effects of Marketability on the Distribution of Market Generated Returns

Approved:

William P. Lane
Major Professor and Chairman

James G. Ingraham
Dean of the Graduate School

EXAMINING COMMITTEE:

William F. Staats

David T. Crary

Charles G. Martin

Donald Marx

Date of Examination: